



Matching in a DSGE Framework

Matthias S. Hertweck

Thesis submitted for assessment with a view to obtaining the degree of
Doctor of Economics of the European University Institute

Florence, January 2010

EUROPEAN UNIVERSITY INSTITUTE
Department of Economics

Matching in a DSGE Framework

Matthias S. Hertweck

Thesis submitted for assessment with a view to obtaining the degree of
Doctor of Economics of the European University Institute

Jury Members:

Prof. Jean-Olivier Hairault, University of Paris I Sorbonne
Prof. Salvador Ortigueira, EUI
Prof. Morten Ravn, University College London, Supervisor
Prof. Antonella Trigari, Università Bocconi

© 2010, Matthias S. Hertweck
No part of this thesis may be copied, reproduced or
transmitted without prior permission of the author

Acknowledgments

The completion of my thesis would not have been possible without the help and support of many people. First of all, I am indebted to my supervisor Morten O. Ravn for his permanent support and encouragement. During the last years, Morten was a constant source of motivation, especially when I had to break through the obstacles involved in writing a PhD thesis. I am also much obliged to my second reader Salvador Ortigueira who gave me the inspiration to focus my research on the macroeconomics of the labor market. Furthermore, I am thankful for many fruitful discussions on labor market issues with my fellow students Clara, Stephan, Georg, and Renato at the European University Institute.

I was introduced to research life during my time as a research assistant at the Halle Institute for Economic Research and Chemnitz University of Technology. In particular, I am very grateful to Karen Pittel, Pia Weiß, Bernhard Eckwert, Thomas Kuhn, Jürgen Meckl, Ralf Müller, Michael Neugart, Michael Pickhardt, and Rainer Voßkamp. Their enthusiasm for economics has stimulated me to start a PhD.

During my PhD studies, I have benefited from financial support by the German Academic Exchange Service, the European University Institute, and the Swiss National Science Foundation (Project No. 118306). Moreover, I enjoyed the hospitality and the organizational support of the European Central Bank during my PhD internship in summer 2008.

I have finalized my thesis holding a postdoctoral research position at the University of Basel. My special thanks go to Aleksander Berentsen and all my colleagues who provided an excellent research environment towards the end of my thesis.

Beyond that, I would like to mention all my friends who have brought joy to my life in Florence. Among others, my former flat-mates Chiara, Christian, and Sebastian; my former office-mates Carolina, Joana, Itai, Konrad, and Sebastian; as well as Afroditi, Eva, Sara, Vera, Alessandro, Christoph, Evangelos, Juan, Patrick, Steven, and Tim.

Finally, I would like to thank my family, my grandmothers, and — most of all — Despina, who gave me love and support during all the years.

I dedicate this thesis to my uncle Winfried who was not granted to defend a thesis.

Contents

0	Introduction	1
1	Strategic Wage Bargaining	3
1.1	Introduction	3
1.2	The Model Environment	6
1.2.1	Labor Market Flows	6
1.2.2	The Problem of the Household	6
1.2.3	The Problem of the Firm	7
1.2.4	The Resource Constraint	9
1.2.5	Wage Determination	9
1.2.6	Optimal Labor Effort	14
1.2.7	Competitive Equilibrium	15
1.3	Model Evaluation	15
1.3.1	Calibration	15
1.3.2	Results	17
1.4	Conclusion	27
	Appendix 1.A Computations	29
1.A.1	Firm's Problem with Linear Vacancy Posting Costs	29
1.A.2	Social Planner Solution	29
1.A.3	The Market Solution is generally not Socially Optimal	30
	Appendix 1.B Tables	32
	Appendix 1.C Figures	34
	Bibliography	36
2	On-the-job Search	41
2.1	Introduction	41
2.2	The Model Environment	43
2.2.1	Employment Relationships	43
2.2.2	The Labor Market	44
2.2.3	The Joint Surplus	45
2.2.4	The Problem of an Unemployed Job Searcher	46

2.2.5	The Problem of an Employed Job Searcher	47
2.2.6	Optimal Vacancy Posting	49
2.2.7	The Wage Function	50
2.2.8	Equilibrium	51
2.3	Model Evaluation	51
2.3.1	Computational Issues	51
2.3.2	Calibration	52
2.3.3	Steady State Analysis	54
2.3.4	Business Cycle Analysis	58
2.4	Conclusion	60
	Appendix 2.A Tables	62
	Appendix 2.B Figures	63
	Bibliography	66
3	Right-to-Manage Wage Bargaining	69
3.1	Introduction	69
3.2	The Model Environment	72
3.2.1	Labor Market Frictions	72
3.2.2	Households	73
3.2.3	The Composite Consumption Good	76
3.2.4	Intermediate Good Firms	76
3.2.5	Employment relations	77
3.2.6	Government and Monetary Authority	79
3.2.7	Market Clearing	80
3.3	Model Evaluation	80
3.3.1	Calibration US	80
3.3.2	Calibration France	82
3.3.3	Inspecting the Mechanism of Staggered Wage Contracts	83
3.3.4	A Transatlantic Perspective	87
3.4	Conclusion	89
	Appendix 3.A The Log-Linear Model	91
	Appendix 3.B Tables	93
	Appendix 3.C Impulse Response Functions	95
	Bibliography	99

Chapter 0

Introduction

This thesis consists of three chapters that examine different aspects of frictional labor markets in a DSGE framework. In particular, my thesis investigates the implications of different wage bargaining regimes, labor adjustment costs, and endogenous on-the-job search. The third chapter represents my contribution to a joint project with Agostino Consolo from the European Central Bank.

The first chapter modifies the standard Mortensen-Pissarides model in order to explain the cyclical behavior of vacancies and unemployment. The modifications include strategic wage bargaining and convex labor adjustment costs. I find that this setup replicates the cyclical behavior of both labor market variables remarkably well. First, I show that the model with strategic wage bargaining matches closely the volatility of vacancies and unemployment. Second, the introduction of convex labor adjustment costs makes both variables much more persistent. Third, my analysis indicates that these two modifications are complementary in generating labor market volatility *and* persistence.

The second chapter addresses the large degree of frictional wage dispersion found in US data. The standard job matching model without on-the-job search cannot replicate this pattern. With on-the-job search, however, unemployed job searchers are more willing to accept low wage offers since they can continue to seek for better employment opportunities. This explains why observably identical workers may be paid very different wages. Therefore, I examine the quantitative implications of on-the-job search in a stochastic job matching model. Our key result is that the inclusion of *variable* on-the-job search increases the degree of frictional wage dispersion by an order of a magnitude.

The third chapter introduces staggered right-to-manage wage bargaining into a New Keynesian business cycle model. My key result is that a reasonably calibrated version of the model is able to generate persistent responses in output, inflation, and total labor input to both neutral technology and monetary policy shocks. Furthermore, I compare the model's dynamic behavior when calibrated to the US and to a European economy. I find that the degree of price rigidity explains most of the differences in response to a monetary policy shock. Differences in the degree of wage rigidity, instead, alter the

dynamics of the model economy only by little. When the economy is hit by a neutral technology shock, both price and wage rigidities turn out to be important. Apart from that, my results indicate that matching frictions matter primarily for the dynamics of the labor market.

Chapter 1

Strategic Wage Bargaining, Labor Market Volatility, and Persistence

1.1 Introduction

The Mortensen-Pissarides job search and matching model¹ has become the standard theory of equilibrium unemployment. Moreover, starting with Merz (1995), Andolfatto (1996) and den Haan et al. (2000), several authors have shown that the inclusion of labor market frictions improves the propagation mechanism of standard real-business-cycle models considerably. Recently, however, the Mortensen-Pissarides model has come under criticism. Following the influential work of Shimer (2005), a large literature has emerged which has shown that the job matching model cannot replicate the cyclical behavior of its two central variables – vacancies and unemployment.

In particular, Shimer (2005) emphasizes that the Mortensen-Pissarides model generates insufficient volatility of vacancies and unemployment at the business cycle frequencies. Indeed, Shimer (2005) challenges not the job search and matching approach itself, but rather the commonly-used Nash (1953) bargaining assumption for wage determination. This approach postulates that the household and the firm divide the mutual surplus period-by-period according to a constant sharing parameter. This implies that the wage bill per worker is almost as elastic as the underlying productivity shock, giving firms only little incentive to adjust the stock of employment. For this reason, Shimer (2005) proposes to consider alternative bargaining assumptions that might deliver real wage rigidity. In a related article, Shimer (2004) provides evidence that real wage rigidity might amplify the volatility of vacancies and unemployment substantially.

Furthermore, Fujita (2004) demonstrates that vacancies in the Mortensen-Pissarides are too less persistent. This artifact follows from firms' hiring behavior in the job matching model with linear vacancy posting costs. In response to a positive technology shock,

¹See Mortensen and Pissarides (1999) as well as Yashiv (2007) for comprehensive surveys.

firms anticipate the sharp and lasting rise in hiring costs and adjust employment instantaneously. Hence, vacancies spike on impact, but fall back half way only one period later. Contrary to this pattern, several authors² have found ample evidence that the impulse response function of vacancies displays a marked hump-shape, peaking with several quarters delay. Fujita and Ramey (2007) address this issue by introducing a sunk cost for the creation of new job positions. This modification improves the persistence of vacancies remarkably. Moreover, the impulse response function of vacancies shows a distinct hump-shape.

The main aim of this paper is to replicate the cyclical behavior of vacancies and unemployment along both dimensions – volatility *and* persistence. Therefore, we modify the standard Mortensen-Pissarides model in two ways. First, we adopt strategic wage bargaining as introduced into the literature by Hall and Milgrom (2008). In contrast to (static) Nash bargaining, strategic wage bargaining assumes that wages are determined by a Rubinstein (1982) game of alternating offers. This approach accounts for the dynamic and interactive character of wage negotiations. The main difference between Nash bargaining and strategic wage bargaining lies in the players' threat points. Under Nash bargaining, both players' threat points are determined by their respective outside alternative, i.e. the value of labor market search. Under strategic wage bargaining, however, the prospective mutual surplus gives both players strong incentives to hold-up the bargaining process until an agreement is reached. Thus, both players' threat points are determined by their respective value of bargaining. As argued by Hall and Milgrom (2008), the value of bargaining is much less sensitive to current labor market conditions than the outside alternative. In our benchmark model, strategic wage bargaining reduces the elasticity of the wage bill per worker by half. As a consequence, the elasticity of the net flow value of the marginal match rises enormously, providing firms much stronger incentives to hire new workers in economic upswings. In this way, strategic wage bargaining gives an endogenous explanation for the observed high degree of labor market volatility.

Second, we combine strategic wage bargaining with convex labor adjustment costs as used by Gertler and Trigari (2009). In contrast to linear vacancy posting costs, firms' hiring costs now are determined by the number of vacancies that are *filled*, and not by the number of vacancies that are *posted*. Further, firms' hiring costs depend negatively on the current employment level. This implies that marginal matching costs are no longer a function of market tightness, but of the gross hiring rate. In contrast to market tightness, the gross hiring rate is much less elastic and much less persistent with respect to technology shocks. The altered behavior of marginal matching costs removes firms' incentives to adjust employment instantaneously. Instead, the convex shape of the labor adjustment cost function gives firms strong incentives to smooth their hiring activities.

²See Blanchard et al. (1989), Fujita (2004), Braun et al. (2006), as well as Ravn and Simonelli (2008), among others.

For this reason, the impulse response function of vacancies in our benchmark model shows a pronounced hump-shape, peaking several quarters after the shock. Consequently, the introduction of convex labor adjustment costs makes vacancies much more persistent, confirming the findings of Yashiv (2006).

Apart from that, we notice that strategic wage bargaining and convex labor adjustment costs are complementary in generating labor market volatility *and* persistence. This interesting result stems from the specification of the hiring cost function. Following Gertler and Trigari (2009), we assume that firms' hiring costs depend negatively on the employment level. Hence, convex labor adjustment costs open a second channel through which strategic wage bargaining amplifies labor market volatility. On the one hand, strategic wage bargaining enhances the volatility of employment by reducing the elasticity of wages. On the other hand, the larger the stock of employment, the lower are the firms' hiring costs. As a result, the introduction of convex labor adjustment costs generates not only more persistence, but also more volatility in the labor market.

Furthermore, we introduce the modified Mortensen-Pissarides model into a real model of the business cycle (Andolfatto, 1996). This seems advantageous, given that this framework allows for a proper calibration of the factor income shares and small (accounting) profits (Hornstein et al., 2005). As demonstrated by Hagedorn and Manovskii (2008), profits have to be small in order to leverage a given productivity shock into large labor market fluctuations. In this context, Mortensen and Nagypál (2007) have shown that the impact of strategic wage bargaining on the volatility of vacancies and unemployment increases considerably once physical capital is considered.

Finally, we find that our setup gives rise to two distortionary effects. In the presence of convex labor adjustment costs, social optimality requires that the wage bill per worker is equal to the household's outside alternative. In contrast, we assume that (i) the wage bill per worker is independent of the fluctuations in household's outside alternative and (ii) firms' bargaining power is smaller than unity. This implies that firms' private gains from labor market search are generally smaller than their social contribution. Consequently, the dynamic behavior of the wage bill per worker is not socially optimal (Hosios, 1990). For this reason, we compute the market solution to our setup, based on the model of Cheron and Langot (2004).

The remainder of this paper is organized as follows. Section 2 presents the model environment. Section 3 calibrates the model and evaluates its quantitative performance against U.S. data. Section 4 concludes.

1.2 The Model Environment

1.2.1 Labor Market Flows

The Mortensen-Pissarides job search and matching model presumes that search on the labor market is frictional. These frictions are represented by a Cobb-Douglas matching function. This function relates aggregate job matches m_t to the number of vacancies that are posted v_t and the search effort of the unemployed $e(1 - n_t)$:

$$m_t(v_t, (1 - n_t)) = \chi v_t^\alpha (e(1 - n_t))^{1-\alpha} \leq \min[v_t, (1 - n_t)], \quad (1.1)$$

where the effort $e > 0$ (“hours”) per unemployed job searcher is taken to be constant. The ratio between vacancies and unemployed job searchers measures the tightness of the labor market. Moreover, we assume that the matching function is linearly homogeneous. Hence, the vacancy filling rate $q(\gamma_t)$ and the job finding rate $q(\gamma_t)\gamma_t$ depend solely on the value of market tightness γ_t :

$$q(\gamma_t) \equiv \frac{m_t}{v_t} = \chi e^{1-\alpha} \left(\frac{(1 - n_t)}{v_t} \right)^{1-\alpha}, \quad (1.2)$$

$$q(\gamma_t)\gamma_t \equiv \frac{m_t}{(1 - n_t)} = \chi e^{1-\alpha} \left(\frac{v_t}{(1 - n_t)} \right)^\alpha. \quad (1.3)$$

These ratios give the expected return on labor market search for firms and the unemployed, respectively. One can observe that the tighter the labor market, the longer the expected time to fill a vacancy, but the shorter the expected search for a job (and vice versa). However, households and firms do not internalize the effect of their search activities on the aggregate return rates. This behavior causes congestion externalities on both market sides.

We assume that new job matches m_t are formed at the end of each period. Simultaneously, a fraction of preexisting jobs is terminated. Consistent with the results of Shimer (2007), we assume the job destruction rate σ to be constant. Consequently, the law of motion for the aggregate employment level is given by:

$$n_{t+1} = (1 - \sigma)n_t + m_t. \quad (1.4)$$

1.2.2 The Problem of the Household

The representative household consists of a continuum of individuals who insure each other completely against idiosyncratic employment risk. The share of employed household members, n_t , works l_t “hours” per period on the job while the share $1 - n_t$ (the unemployed) searches e “hours” on the labor market. Both activities affect utility negatively as they

reduce the amount of leisure. We assume the following per period utility function:

$$\begin{aligned} u^N(c_t^N, 1 - l_t) &= \ln(c_t^N) + \phi_1 \frac{(1 - l_t)^{1-\eta}}{1 - \eta}, \\ u^U(c_t^U, 1 - e) &= \ln(c_t^U) + \phi_2 \frac{(1 - e)^{1-\eta}}{1 - \eta}. \end{aligned}$$

The parameter $\phi_i, i = 1, 2$ captures the fact that the valuation of leisure depends on the employment status. Each employed household member earns the real wage rate w_t per hour l_t . Hence, $n_t w_t l_t$ constitutes the labor income of the representative household. In addition, households receive dividends remitted by firms π_t and rental income $r_t k_t$ from perfectly competitive capital markets. The state space of the household is given by the set $\Omega_t^H = \{k_t, n_t\}$. Thus, the maximization problem of the representative household can be formulated as:

$$\begin{aligned} \mathcal{W}(\Omega_t^H) &= \max_{c_t^U, c_t^N, k_{t+1}} \left\{ n_t u^N(c_t^N, 1 - l_t) + (1 - n_t) u^U(c_t^U, 1 - e) + \beta E_t [\mathcal{W}(\Omega_{t+1}^H)] \right\}, \\ \text{s.t.} & \end{aligned} \tag{1.5}$$

$$k_{t+1} = (1 - \delta + r_t)k_t + \pi_t + n_t w_t l_t - n_t c_t^N - (1 - n_t) c_t^U, \tag{1.6}$$

$$n_{t+1} = (1 - \sigma)n_t + q(\gamma_t)\gamma_t(1 - n_t). \tag{1.7}$$

Here, equation (1.6) is the budget constraint. Equation (1.7) is the law of motion for the household's employment share. Provided stochastic processes for $\{w_t, r_t, l_t, \pi_t, q(\gamma_t)\gamma_t \mid t \geq 0\}$ and a set of initial conditions $\{k_0, n_0\}$, the representative household chooses contingency plans $\{c_t^U, c_t^N, k_{t+1} \mid t \geq 0\}$ that maximize its expected discounted utility. These choices have to satisfy following first order conditions:

$$c_t^N : \lambda_t = u_1^N(c_t^N, 1 - l_t), \tag{1.8}$$

$$c_t^U : \lambda_t = u_1^U(c_t^U, 1 - e), \tag{1.9}$$

$$k_{t+1} : \lambda_t = \beta E_t[\lambda_{t+1}(1 - \delta + r_{t+1})]. \tag{1.10}$$

The first order conditions (1.8) and (1.9) show that perfect income insurance against idiosyncratic employment risk allocates the same consumption level to employed and unemployed workers. Equation (1.10) gives the familiar Euler equation for consumption.

1.2.3 The Problem of the Firm

Output is produced by firms that use capital k_t and labor hours ($n_t l_t$) as input factors. The production function is taken to be Cobb-Douglas with constant returns to scale. This implies that the model has a representative firm. We assume that total factor productivity

a_t is subject to an exogenous shock specified by the following autoregressive process:

$$\ln(a_t) = (1 - \rho) \ln(\bar{a}) + \rho \ln(a_{t-1}) + \epsilon_t, \text{ with } \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2) \text{ and } iid. \quad (1.11)$$

The specification of the firm's cost function follows Gertler and Trigari (2009). The firm incurs rental costs of capital $r_t k_t$, aggregate wage payments $n_t w_t l_t$, and labor adjustment costs ψ_t :

$$\psi(m_t, n_t) = \frac{\kappa}{2} \frac{m_t^2}{n_t} \quad (1.12)$$

In contrast to the standard specification,³ labor adjustment costs are determined by the squared number of new job matches m_t^2 , the employment level n_t , and a constant scale parameter $\kappa/2$.⁴ Consequently, firms' labor adjustment costs are determined by the number of vacancies that are *filled*, and not by the number of vacancies that are *posted*. In other words, vacancy posting *per se* is costless. In addition, notice that firms take the aggregate vacancy filling rate $q(\gamma_t)$ as given. Hence, from the perspective of the representative firm, the number of new job matches $m_t = q(\gamma_t)v_t$ is linear in vacancies.

Moreover, we assume that the representative firm is large in the sense that it has many workers, and that it is large enough to eliminate all uncertainty about n_{t+1} . This ensures that all firms in the model remain of the same size (Rotemberg, 2006). However, the representative firm in our model is small in the sense that it is competitive. For this reason, the firm takes not only the aggregate vacancy filling rate, but also the wage bill per worker, $w_t l_t$, as given.⁵ The state space of the firm is given by the set $\Omega_t^F = \{n_t\}$. Thus, the representative firm's problem can be formulated as:

$$\mathcal{V}(\Omega_t^F) = \max_{k_t, v_t} \left\{ y_t - n_t w_t l_t - r_t k_t - \frac{\kappa}{2} \frac{m_t^2}{n_t} + \beta E_t \left[(\lambda_{t+1}/\lambda_t) \mathcal{V}(\Omega_{t+1}^F) \right] \right\}, \quad (1.13)$$

s.t.

$$y_t = a_t k_t^\theta (n_t l_t)^{(1-\theta)}, \quad (1.14)$$

$$n_{t+1} = (1 - \sigma)n_t + q(\gamma_t)v_t. \quad (1.15)$$

Given stochastic processes for $\{a_t, w_t, r_t, l_t, q(\gamma_t) \mid t \geq 0\}$ and an initial condition for n_0 , the representative firm chooses contingency plans $\{k_t, v_t, n_{t+1} \mid t \geq 0\}$ that maximize the

³See Appendix 1.A.1 for the firm's problem with linear vacancy posting costs $\psi(v_t) = \kappa v_t$.

⁴Several recent studies (Nilsen et al., 2007; King and Thomas, 2006; Merz and Yashiv, 2007) provide evidence for the empirical relevance of convex adjustment cost functions at the macro level.

⁵In words, the representative firm does not internalize the impact of its hiring activities on the expected wage bill per worker $(\partial w_{t+1} l_{t+1})/(\partial n_{t+1}) = 0$. Nevertheless, the firm anticipates the future wage bill per worker $w_{t+1} l_{t+1}$ correctly. See Section 1.2.5 for more information.

expected present value of the dividend flow. The first order conditions are given as:

$$k_t : r_t = \theta \frac{y_t}{k_t}, \quad (1.16)$$

$$n_{t+1} : \kappa x_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left((1 - \theta) \frac{y_{t+1}}{n_{t+1}} - w_{t+1} l_{t+1} + \frac{\kappa}{2} x_{t+1}^2 + (1 - \sigma) \kappa x_{t+1} \right) \right], \quad (1.17)$$

where the gross hiring rate m_t/n_t is denoted by x_t . Equation (1.16) shows the familiar relation between the real interest rate and the marginal product of capital under perfectly competitive capital markets. The hiring condition (1.17) states that the representative firm posts the optimal number of job vacancies v_t that equalizes expected marginal hiring costs κx_t (the left hand side) with the expected present value of the marginal match in the future (the right hand side). The expected present value of the marginal match depends on the marginal product per worker $(1 - \theta)(y_{t+1}/n_{t+1})$, the expected wage bill per worker $w_{t+1} l_{t+1}$, expected savings on *adjustment* costs $(\kappa/2)x_{t+1}^2$, and expected savings on *hiring* costs $(1 - \sigma)\kappa x_{t+1}$. Savings on *adjustment* costs capture the fact that each marginal match increases the stock of employment in the next period, irrespective of when the match is terminated. On the contrary, savings on *hiring* costs are only realized if the match survives the following period.

1.2.4 The Resource Constraint

The following equation gives the resource constraint of our economy. The resource constraint postulates that output is divided into consumption, gross investment and labor adjustment costs:

$$y_t = c_t + k_{t+1} - (1 - \delta)k_t + \frac{\kappa}{2} \frac{m_t^2}{n_t}. \quad (1.18)$$

1.2.5 Wage Determination

The Bargaining Set

Frictions in the labor market create a prospective mutual surplus between firm-worker matches. This surplus equals the value added of the match compared to the payoff of both parties in the labor market. Following Pissarides (2000, chapter 3), we assume that the wage bill per worker $w_t l_t$ is determined for each match separately while wages in all other matches are taken as given. Hence, the relevant surplus share of the household and the firm, respectively, is determined by the marginal job match:

$$\begin{aligned} \mathcal{W}_2(\Omega_t^H) &= \left\{ \lambda_t (w_t l_t + c_t^U - c_t^N) + (1 - \sigma) \beta E_t [\mathcal{W}_2(\Omega_{t+1}^H)] \right\} - \\ &\quad \left\{ u^U(c_t^U, 1 - e) - u^N(c_t^N, 1 - l_t) + q(\gamma_t) \gamma_t \beta E_t [\mathcal{W}_2(\Omega_{t+1}^H)] \right\}, \end{aligned} \quad (1.19)$$

$$\mathcal{V}_1(\Omega_t^F) = (1 - \theta) F_{2,t} l_t - w_t l_t + \frac{\kappa}{2} x_t^2 + (1 - \sigma) \beta E_t \left[(\lambda_{t+1}/\lambda_t) \mathcal{V}_1(\Omega_{t+1}^F) \right]. \quad (1.20)$$

The surplus share of the household $\mathcal{W}_2(\Omega_t^H)$ equals the difference between the value of employment and the value of unemployment.⁶ The value of employment is made up of the sum of the wage bill per worker and household's expected present value of the match in the future. The value of unemployment, i.e. household's outside alternative, consists of the current utility gain from leisure and household's continuation payoff from labor market search. The surplus of the firm $\mathcal{V}_1(\Omega_t^F)$ is composed of (i) the marginal product per worker, (ii) the wage bill per worker, (iii) savings on *adjustment* costs, and (iv) the expected present value of the match in the future. A non-arbitrage condition ensures that the outside alternative of the firm (i.e. the ex-ante value of an unfilled vacancy) is zero. The sum of the marginal product per worker, i.e. the marginal product per "hour" $F_{2,t}$ times "hours worked" h_t , and savings on adjustment costs $(\kappa/2)x_t^2$ is defined as gross flow value of the marginal match. Given that the weight of the marginal match is small, both parties take the gross flow value of the marginal match as given during the bargaining process.

Thus, the mutual surplus S_t of the marginal firm-worker match (in units of the consumption good) is given as the sum of the two shares:

$$S_t = (\mathcal{W}_2(\Omega_t^H)/\lambda_t) + \mathcal{V}_1(\Omega_t^F). \quad (1.21)$$

The allocation of the mutual surplus between the household and the firm determines the wage bill per worker $w_t l_t$. In order to satisfy individual rationality, the equilibrium wage bill per worker has to make each party at least indifferent between accepting the contract and the forgone outside alternative of continued labor market search. We obtain the reservation wage bill (per worker) of the household and the firm, respectively, by setting the surplus share equal to zero. Equation (1.19) shows that the reservation wage bill of the household $(wl)_{min}$ is given by the value of unemployment *less* household's expected value of the match in the future:

$$(wl)_{min} = \frac{1}{\lambda_t} \left\{ u(c_t^U, 1-e) - u(c_t^N, 1-l_t) + q(\gamma_t) \gamma_t \beta E_t [\mathcal{W}_2(\Omega_{t+1}^H)] - (1-\sigma) \beta E_t [\mathcal{W}_2(\Omega_{t+1}^H)] \right\}. \quad (1.22)$$

Analogously, the reservation wage bill of the firm $(wl)_{max}$ is defined as the gross flow value of the marginal match *plus* firm's expected present value of the marginal match in the future:⁷

$$(wl)_{max} = (1-\theta)F_{2,t}l_t + \frac{\kappa}{2}x_t^2 + (1-\sigma)\beta E_t [(\lambda_{t+1}/\lambda_t)\mathcal{V}_1(\Omega_{t+1}^F)]. \quad (1.23)$$

⁶With perfect insurance against unemployment, the level of consumption is independent of the employment status ($c_t^N = c_t^U$).

⁷Note that firms treat hiring costs as sunk. Hence, a firm would generate negative profits if it accepted a wage bill per worker close to $(w_t l_t)_{max}$ (Hall and Milgrom, 2008). However, this possibility is ruled out by our calibration.

These two reservation wage bills constitute the lower and the upper bound of the bargaining set which contains all feasible wage bills (Malcomson, 1999). In other words, the equilibrium value of the wage bill per worker is indeterminate. Therefore, we assume that wages are determined by an *ex-post* bargaining game between the household and the firm. In particular, we consider two alternative approaches – standard Nash (1953) bargaining and a Rubinstein (1982) game of alternating offers. In addition, the wage bill per worker is subject to continuous renegotiation whenever new information arrives. In our discrete-time model, this implies that new matches are formed at the end of each period. However, bargaining does not start until the beginning of the next period when the new state of technology can be observed.

The Optimal Wage Contract

For the standard job search and matching model (with linear vacancy posting costs), Hosios (1990) has established a necessary and sufficient condition under which both congestion externalities just offset one another.⁸ As mentioned above, the congestion externalities arise from the fact that firms take the aggregate vacancy filling rate $q(\gamma_t)$ as given when deciding upon the optimal number of vacancies v_t . Thus, from the firm's perspective, the number of new job matches $m_t = q(\gamma_t)v_t$ is linear in vacancies. Accordingly, the firm's private gain of the marginal vacancy is given as (see Appendix 1.A.1):

$$\kappa = q(\gamma_t)\mathcal{V}_1(\Omega_t^F). \quad (1.24)$$

In contrast, the social planner solution accounts for the fact that new job matches are a concave function of vacancies (see equation 1.1). Hence, the social planner internalizes that the vacancy filling rate decreases in the number of posted vacancies. Therefore, the marginal vacancy yields following social benefit (see Appendix 1.A.2):

$$\kappa = \alpha q(\gamma_t)S_t. \quad (1.25)$$

Social optimality requires that firms' private gains from search effort equals their social benefit. Consequently, firms' incentives to post vacancies are efficient if and only if

$$\mathcal{V}_1(\Omega_t^F) = \alpha S_t \quad (1.26)$$

holds. In words, the Hosios condition postulates that the private gain per match equals the share α of the mutual surplus S_t per match.

In contrast, if firms internalized the congestion effect on the aggregate vacancy filling rate correctly, social optimality would require firms to gain the entire mutual surplus per

⁸Merz (1995) has generalized the Hosios condition for dynamic models.

match, i.e.:

$$\mathcal{V}_1(\Omega_t^F) = S_t. \quad (1.27)$$

However, if firms gained the entire mutual surplus, even though they did not internalize the congestion effects, the private gains from the marginal vacancy would be larger than the social benefit. Thus, firms would have an incentive to “over-hire”. In order to avoid the over-hiring effect, the Hosios condition requires that firms gain only the share α of the mutual surplus per match.

Under convex labor adjustment costs, on the contrary, firms’ hiring costs depend on the number of vacancies that are filled $q(\gamma_t)v_t$, and not on the number of vacancies that are posted v_t . Consequently, the congestion externalities bias not only firms’ private gains, but also – to the same extent – firms’ hiring costs. This removes firms’ incentives to over-hire, even if they gained the entire mutual surplus per match. Under these circumstances, the congestion externalities exactly offset each other if and only if the entire mutual surplus per match accrues to the firms (see Appendix 1.A.3), i.e. if equation (1.27) holds.

Nash Bargaining

Nash bargaining has become the standard method for wage determination in job matching models. This approach postulates a number of axioms and derives a unique equilibrium sharing rule for the mutual surplus. In addition, Nash (1953) proves that exactly the same solution can be generated by a simultaneous one-shot game. This bargaining game presumes that both parties threaten each other to terminate the bargain unilaterally rather than to conclude an agreement. Subsequently, both parties reveal their demands simultaneously. If these demands are not compatible, the match is broken up and both players gain only their respective outside alternative, i.e. they return to labor market search. However, given perfect information and rational players, Nash (1953) shows that both parties agree on following unique sharing rule:

$$w_t l_t = \arg \max_{w_t l_t} \left\{ \left(\mathcal{W}_2(\Omega_t^H)/\lambda_t \right)^{1-\xi} \left(\mathcal{V}_1(\Omega_t^F) \right)^\xi \right\}, \quad (1.28)$$

where the original version assumes symmetric bargaining power ($\xi = 1/2$). The generalized version, however, allows any value for ξ in the interval $(0, 1]$.⁹ Hence, the solution to our model is given by the wage bill per worker which maximizes the weighted product of both parties’ surplus shares. This sharing rule allocates period-by-period a constant share of the mutual surplus to each of the two parties:

$$\xi \left(\mathcal{W}_2(\Omega_t^H)/\lambda_t \right) = (1 - \xi) \mathcal{V}_1(\Omega_t^F). \quad (1.29)$$

Thus, in the case of linear vacancy posting costs, the Nash solution generates the

⁹Note that equation 1.30 and equation 1.31 are not defined for $\xi = 0$.

socially optimal bargaining outcome if and only if firms' bargaining power ξ coincides with the matching elasticity of vacancies α . With convex labor adjustment costs, however, social optimality requires that the entire match surplus S_t accrues to the firms (i.e. $\xi = 1$). This implies that the wage bill per worker $w_t l_t$ is equal to the household's outside alternative. Nevertheless, we consider the general case $\xi \in (0, 1]$ throughout our analysis.

The resulting wage bill per worker equals the weighted average of the gross flow value of the marginal match and household's outside alternative:

$$w_t l_t = (1 - \xi) \left[(1 - \theta) \frac{y_t}{n_t} + \frac{\kappa}{2} x_t^2 \right] + \xi \left[\frac{u_t^U - u_t^N}{\lambda_t} + \frac{(1 - \xi)}{\xi} \frac{m_t \kappa x_t}{(1 - n_t)} \right]. \quad (1.30)$$

Household's outside alternative depends on the flow value of unemployment, i.e. the current utility gain in leisure $(u_t^U - u_t^N)/\lambda_t$, and the continuation payoff from labor market search. The latter, in turn, depends on the current job finding rate $m_t/(1 - n_t)$ times her adjusted share $(1 - \xi)/\xi$ of the expected present value of a *prospective* future match κx_t . Consequently, household's outside alternative is very sensitive to current labor market conditions.

Notably, the expected present value of the *current* match (see equation 1.19 and equation 1.20) does not enter equation (1.30). This is due to the fact that the mutual surplus is always allocated according to the same sharing rule (1.28). Hence, both expressions widen the bargaining set proportionally, but have no impact on the bargaining outcome. We define the replacement rate b as the ratio between the flow value of unemployment and the gross flow value of the marginal match.

Strategic Wage Bargaining

Hall and Milgrom (2008) highlight that Nash bargaining abstracts from the dynamic and interactive character of wage negotiations. For that reason, they argue that wages in the job matching model should be determined by a Rubinstein (1982) game of alternating offers. In particular, Hall and Milgrom (2008) emphasize the crucial importance of the prospective mutual surplus. The mutual surplus gives both players strong incentives to conclude the bargaining successfully. Hence, neither party seriously considers breaking up the bargaining process completely. Given perfect information, this implies that threatening to terminate the bargaining process is not a credible option (Schelling, 1960). Instead, both parties threaten each other to reject unfavorable demands. Since both parties are impatient, this strategy causes costly delays and gives them the incentive always to make acceptable demands. Consequently, once a firm-worker match has successfully been formed, it is the value of bargaining – and not the outside alternative – that determines the relevant surplus.

In their analysis, Hall and Milgrom (2008) focus on the limiting case in which the time interval between successive offers decreases to zero. Under these circumstances, they

show that both parties agree on the equilibrium wage bill per worker instantaneously. This allows us to approximate the solution to the dynamic bargaining game by a corresponding static game (Binmore et al., 1986). The solution to this new game can be found by maximizing the weighted product of the two surplus shares – like in the standard Nash solution. However, the solution to this dynamic bargaining problem is inherently different from the Nash solution as the surplus of each party is no longer determined by the respective outside alternative, but by the losses associated with delays.

Following Hall and Milgrom (2008), we calibrate the dynamic bargaining model to the same steady state as the standard bargaining model. This simplifying assumption implies that the steady state value of bargaining coincides with the outside alternative.¹⁰ Furthermore, Hall and Milgrom (2008) emphasize that the value of bargaining might depend less sensitively on *current* labor market conditions than the outside alternative. Thus, they take the value of bargaining to be time-invariant. For this reason, we replace all variables in equation (1.30) that derive from the outside alternative with their steady state values (denoted by an over line):

$$w_t l_t = (1 - \xi) \left[(1 - \theta) \frac{y_t}{n_t} + \frac{\kappa}{2} x_t^2 \right] + \xi \left[\frac{\bar{u}^U - \bar{u}^N}{\bar{\lambda}} + \frac{(1 - \xi)}{\xi} \frac{\bar{m} \kappa \bar{x}}{(1 - \bar{n})} \right]. \quad (1.31)$$

This sharing rule is equivalent to Nash bargaining with a constant outside alternative. Given that the outside alternative is typically pro-cyclical, the dynamic bargaining game generates a less elastic wage bill per worker than Nash bargaining. Consequently, the households' share of the surplus falls below $(1 - \xi)$ in economic upswings (and vice versa). Note that the wage bill per worker satisfies individual rationality as long as it remains within the bargaining set.

In summary, strategic wage bargaining gives rise to two distortionary effects. As discussed above, social optimality under convex labor adjustment costs requires that the wage bill per worker $w_t l_t$ is equal to the household's outside alternative. In contrast, we assume that (i) the wage bill per worker is independent of the fluctuations in household's outside alternative and (ii) firms' bargaining power ξ is generally smaller than unity, i.e. $\xi \in (0, 1]$. Hence, firms' private gains from search effort are generally smaller than their social contribution. In this case, the dynamic behavior of the wage bill per worker is not socially optimal (Hosios, 1990).

1.2.6 Optimal Labor Effort

The model is closed with the condition for optimal labor effort l_t ("hours"). We assume that both parties have a joint interest to maximize the value of the mutual surplus S_t .

¹⁰Actually, if the value of bargaining coincided with the outside alternative, the respective player would be indifferent between delaying and terminating the bargain.

Provided that the marginal product per “hour” $F_{2,t}$ is taken as given by both parties, the maximization problem of S_t with respect to l_t yields following condition:

$$(1 - \theta) \frac{y_t}{n_t l_t} = \frac{1}{\lambda_t} \frac{\phi_1}{(1 - l_t)^\eta}. \quad (1.32)$$

This condition determines how the wage bill per worker is split up into the real wage rate per “hour” w_t and “hours” per worker l_t .

1.2.7 Competitive Equilibrium

The competitive equilibrium is a set of allocations $\{c_t, k_{t+1}, v_t, n_{t+1}\}$ and prices $\{r_t, w_t\}$, such that:

- (i) employment relationships are governed by the matching function (1.1) and the law of motion of employment (1.4)
- (ii) $\{c_t, k_{t+1}\}$ solves the household’s problem (1.5) subject to the budget constraint (1.6) and the law of motion for its employment share (1.7)
- (iii) total factor productivity follows the exogenous stochastic process (1.11)
- (iv) $\{k_t, v_t\}$ solves the firm’s problem (1.13) subject to the production technology (1.14) and the law of motion for its stock of employment (1.15)
- (v) the resource constraint (1.18) holds and the perfectly competitive capital market clears
- (vi) the wage bill per worker is determined either by Nash bargaining (1.30) or by strategic wage bargaining (1.31)
- (vii) hours per worker maximize the mutual surplus (1.32)
- (viii) an initial condition for the state space (k_0, n_0, z_0) is given

Consequently, the competitive equilibrium is defined by following conditions: (1.1), (1.4), (1.8), (1.9), (1.10), (1.11), (1.14), (1.16), (1.17), (1.18), either (1.30) or (1.31), and (1.32).

1.3 Model Evaluation

1.3.1 Calibration

We calibrate the model so that one period corresponds to a month. This seems advantageous given that, in the U.S., the job finding rate is very high. When we simulate the model, we time-aggregate the artificial data to quarterly frequencies in order to make

them comparable to the U.S. aggregate time series. Table 1.1 summarizes the parameter values of our model.

Using data on aggregate income shares, Cooley and Prescott (1995) calibrate the production elasticities of capital ($\theta = 0.40$) and labor ($1 - \theta = 0.60$). We adopt their conventional values, even though the production elasticity of labor is slightly larger than the average labor share in our job matching model (0.58, see table 1.2). In addition, we set the monthly depreciation rate δ to match an annual rate of 10% (Kydland and Prescott, 1982).

β is chosen to be consistent with a quarterly real interest rate of 1 percent. Following Juster and Stafford (1991), we set the steady state working time of employed household members to $\bar{l} = 1/3$ of their discretionary time endowment. Moreover, Barron and Gilley (1981) estimate that the typical unemployed primarily engaged in random job search (approximately one half of the sample) spends between 8 and 9 hours per week to contact potential employers. This corresponds to about 25% of the average working time \bar{l} . For the given specification of preferences (Andolfatto, 1996), the elasticity of intertemporal substitution in labor supply is given as: $\nu = \eta^{-1}((1/\bar{l}) - 1)$. Blundell and Macurdy (1999) provide robust evidence that the value of ν for annual hours of employed men is between 0.1 and 0.3. For *employed* women, Blundell et al. (1988) and Triest (1990) estimate values in the same range. However, Browning et al. (1999) observe that leisure is more substitutable over shorter intervals than longer ones. Using monthly data on employed men, MaCurdy (1983) finds significantly higher elasticities (0.3 – 0.7). Hence, we choose ν equal to 0.5, which implies setting $\eta = 4$.

We calibrate the monthly job separation rate to 3.5% (Shimer, 2007). This value implies that the average job duration is 2 1/2 years. Furthermore, the steady state unemployment rate is set to 10 percent (Hall, 2005b). This measure includes the officially unemployed job searchers and the pool of marginally attached non-participants (Jones and Riddell, 1999). Thus, our calibration implies that the average job finding rate, $q(\bar{\gamma})\bar{\gamma}$, is equal to 0.32 (see table 1.2), which is consistent with the results of Hall (2005b). The monthly vacancy filling rate is set to match the quarterly value $q(\bar{\gamma}) = 0.71$ estimated by van Ours and Ridder (1992).¹¹ Based on the fact that per-period labor adjustment costs “are not much more than one percent of per-period payroll cost” (Hamermesh and Pfann, 1996, p. 1278), we calibrate aggregate labor adjustment costs $\bar{\psi}$ equal to 1% of aggregate output. This value implies that the average replacement ratio \bar{b} is equal to 63%. This value is somewhat larger than the upper bound ($\bar{b} = 40\%$) estimated by Shimer (2005). However, Shimer (2005) interprets b entirely as an unemployment benefit. In our model b includes also utility costs of working, e.g. leisure value of unemployment or the value of home production (Hagedorn and Manovskii, 2008). Un-

¹¹According to (Shimer, 2005), the model allows the normalization of the vacancy filling rate. Nevertheless, we choose a meaningful value.

fortunately, empirical evidence on the size of the leisure value of unemployment is scarce (Holmlund, 1998). According to Costain and Reiter (2008), the upper bound of the utility costs of working is equal to 75%. Hence, our value seems reasonable.¹²

We calibrate the matching elasticity of unemployment to $\alpha = 0.5$. This value is within the plausible range (0.5 – 0.7) proposed by Petrongolo and Pissarides (2001). In addition, we assume symmetrically distributed bargaining power, i.e. $\xi = 0.5$ (Svejnar, 1986). Thus, as mentioned above, our model gives rise to two distortionary effects. On the one hand, we assume that the wage bill per worker is independent of the fluctuations in the household's outside alternative. On the other hand, firms' bargaining power ξ is strictly smaller than unity.¹³ Consequently, their private gains from search effort are generally smaller than their social contribution (Hosios, 1990). Nevertheless, we set $\alpha = \xi$ in order to facilitate comparison with the existing literature.

We calibrate the law of motion for the technology shock by setting the monthly autocorrelation coefficient ρ equal to 0.9830 and the standard deviation σ_ϵ equal to 0.0044. The monthly autocorrelation coefficient is chosen to match the conventionally used quarterly value of 0.95 (Cooley and Prescott, 1995). Furthermore, we set the standard deviation of the monthly process so that the volatility of the time-aggregated Solow residual is in accordance with a standard-calibrated quarterly real business cycle model (Cooley and Prescott, 1995).¹⁴

Notice that our calibration ensures that the reservation value of the firm $(\overline{wl})_{max}$ is larger than the reservation value of the household $(\overline{wl})_{min}$.

1.3.2 Results

This section examines the quantitative performance of the modified job matching model. We analyze how the chosen wage determination mechanism and the costs of labor adjustment, respectively, affect the dynamics of the model in response to technology shocks. Moreover, we highlight the interactions between both modifications.

We evaluate the model generated time series against quarterly U.S. data from 1964:1 to 1999:4. Most of the time series are from the Federal Reserve Bank of St. Louis (FRED®). In addition, we use the expanded unemployment series from Hall (2005b). From this data we construct a set of time series which corresponds to the variables in our model (see table 1.3 and table 1.4). We log and detrend all series using the Hodrick and Prescott

¹²In the model with linear vacancy posting costs, $\overline{\psi} = 0.01$ implies that the average replacement ratio is equal to 81%. This value is slightly larger than the upper bound ($\bar{b} = 75\%$) suggested by Costain and Reiter (2008), but still below the estimate $\bar{b} = 94\%$ of Hagedorn and Manovskii (2008). The choice for \bar{b} is crucial, because a larger b decreases the surplus. Consequently, the higher the value of \bar{b} , the easier it is to leverage a given shock into labor market fluctuations.

¹³The social planner's problem is documented in Appendix 1.A.2.

¹⁴Cooley and Prescott (1995) estimate the quarterly parameters ($\rho = 0.95, \sigma_\epsilon = 0.007$) assuming that the labor income share equals $1 - \theta$. In labor search models, this assumption holds only as an approximation.

(1997) filter assuming a smoothing parameter of 1600.

Table 1.5 reports the well-known business cycle statistics of the U.S. labor market. In particular, we focus on the cyclical behavior of vacancies v and unemployment $1 - n$. The data reveal that both variables are highly volatile and very persistent. In addition, vacancies are clearly pro-cyclical whereas unemployment is strongly counter-cyclical. Consequently, vacancies and unemployment are almost perfectly negatively correlated ($\rho_{vU} = -0.95$). Due to the strong persistence of both variables, we observe that the negative correlation between vacancies and unemployment remains also at leads and lags (Fujita, 2004). Hence, the dynamic correlation structure between vacancies and unemployment follows a pronounced U-shape (see table 1.6 and figure 1.3). This pattern is known as the “dynamic Beveridge curve”.¹⁵ Furthermore, we observe that the wage bill per worker wl is significantly less volatile and less pro-cyclical than output per worker y/n .

We log-linearize the model around the non-stochastic steady state and solve for the recursive law of motion using the “Toolkit” from Uhlig (1998).¹⁶ Corresponding to the U.S. data sample period, we simulate the model to 432 “monthly” data points. Subsequently, we transform the artificial data as described above and compute the statistics over 10.000 simulations.

Comparative Impulse Response Analysis

We now inspect the model’s impulse responses to a one percent shock in total factor productivity. In particular, we explore the role of the chosen wage determination mechanism and the costs of labor adjustment, as well as the interactions between them. Figure 1.1 compares the impulse responses of the strategic bargaining model with convex labor adjustment costs (henceforth called the “benchmark model”, denoted by a solid line), the Nash bargaining model with convex labor adjustment costs (henceforth called the “NB model”, denoted by a dashed line), and the and the strategic wage bargaining model with linear vacancy posting costs (henceforth called the “LC model”, denoted by a dotted line). The graphs depict the evolution of the relevant variables over 96 months (32 quarters).

The Benchmark Model The hiring condition (1.17) reveals that the pattern of cyclical employment adjustment is governed by two main determinants: First, the net flow value of the marginal match captures cyclical variations in the return to additional employment. Second, the structure of labor adjustment costs determines how fast and at what cost firms adjust employment over the business cycle. In the following, we focus on the impact of these two factors.

¹⁵See, inter alia, Fujita and Ramey (2006) and the references therein.

¹⁶The above calibration ensures a unique and stable equilibrium.

In response to a one percent technology shock, we observe that the gross flow value of the marginal match rises by about one percent. The elasticity of the wage bill per worker, in contrast, is significantly lower. This follows directly from strategic wage bargaining (see equation 1.31). Accordingly, the elasticity of the wage bill per worker is given by the elasticity of the gross flow value of the marginal match times the household's bargaining power $(1 - \xi)$. As a result, the costs per worker increase much less than the gains. This generates an increase of about 17 percent in the net flow value of the marginal match, giving firms strong incentives to amplify employment adjustment over the business cycle.

In the case of convex labor adjustment costs, firms choose the optimal number of vacancies v_t such that expected marginal matching costs κx_t are equal to the expected present value of the marginal match. Due to the convex shape of ψ_t , new job matches m_t are much less elastic than the net flow value of the marginal match. Furthermore, the convex shape of ψ_t gives firms strong incentives to smooth hiring activities over several periods. For this reason, new job matches rise on impact by somewhat more than 4 percent and then remain well above their steady state value for the entire observation period. This continuous inflow of new job matches leads to a pronounced hump-shape in the impulse response function of employment, which peaks about 2 1/2 years after the shock. Consequently, the impulse response function of unemployment follows a distinct U-shape.

Moreover, the strong reaction in employment feeds back to the expected marginal matching costs. Given that employment is a state variable, the impulse response function of expected marginal hiring costs increases on impact by exactly the same amount as new job matches. In the following periods, however, the long-lasting increase in employment dampens marginal hiring costs. Hence, the impulse response function of marginal hiring costs converges relatively quickly to its steady state value. This pattern reinforces gradual and long-lasting hiring activities and, thus, might explain the remarkable slow convergence of new job matches.

Finally, we analyze the impulse response function of vacancies. As mentioned above, we assume that firms' hiring costs depend on the number of vacancies that are *filled*, and not on the number of vacancies that are *posted*. Therefore, firms always post the number of vacancies that is necessary to obtain the optimal number of new job matches. According to the aggregate matching function (see equation 1.1), the number of new job matches is given by the current level of unemployment and the number of vacancies that are posted. In response to a positive technology shock, firms face following scenario: On the one hand, firms have to maintain a continuous inflow of new job matches. On the other hand, the impulse response function of unemployment decreases sharply over more than 2 1/2 years. This leads to a strong fall in the vacancy filling rate. The lower the vacancy filling rate, the more vacancies have to be posted in order to obtain the optimal number of new matches. For this reason, the impulse response function of vacancies increases on

impact by about 10 percent. In the following periods, vacancies continue to rise and reach a maximum of 18 percent increase with 2 1/2 years delay. In words, the impulse response of vacancies follows a marked hump-shape. This pattern is found to be consistent with the data.¹⁷

The Impact of Strategic Wage Bargaining We now discuss the impulse responses of the “NB model”. Under Nash bargaining, the elasticity of the wage bill per worker is not only determined by the gross flow value of the marginal match, but also by the household’s outside alternative. Given that household’s outside alternative is clearly pro-cyclical, we note that the elasticity of the wage bill per worker increases substantially. Hence, the wage bill per worker is nearly as elastic as the gross flow value of the marginal match. As a result, the costs per worker increase almost as much as the gains. This implies that the elasticity of the net flow value of the marginal match decreases enormously. Additionally, due to the hump-shape in the household’s outside alternative, the net flow value of the marginal match is less persistent than in the benchmark model. This illustrates that Nash bargaining gives firms much less incentives to hire new workers than strategic wage bargaining.

In fact, we observe that firms’ hiring activities decline dramatically. On impact, new job matches rise only by less than one percent and then fall back quickly to their steady state value. Thus, the impulse response of employment is substantially smaller. For the same reason, the U-shaped response of unemployment is much weaker. Moreover, due to the mild response of employment, the feedback effect from employment on lower expected marginal matching costs is almost not present.

The modest increase in matches, in conjunction with the weak reduction in unemployment, implies that the vacancy filling rate reduces only slightly. Consequently, vacancies rise on impact only by somewhat more than one percent, continue to increase slightly for about 3 quarters, and then return slow and monotone to their steady state value. For this reason, vacancies are much less elastic than in the benchmark model. Furthermore, we observe that the hump-shaped dynamics of the impulse response functions are less distinct.

We conclude that strategic wage bargaining amplifies the elasticity of employment, unemployment and vacancies enormously. Apart from that, the hump-shaped (U-shaped) response of vacancies (unemployment) is more distinct under Nash bargaining.

The Impact of Convex Labor Adjustment Costs We now examine the impact of convex labor adjustment costs on the dynamic behavior of the labor market. Therefore, we compare the impulse responses of the benchmark model with the impulse responses of the “LC model”. In both cases under consideration, the wage bill per worker is determined

¹⁷See Footnote 2 and the references therein.

by strategic wage bargaining. We observe that the instantaneous elasticities of the gross flow value of the marginal match, household's outside alternative, and the wage bill per worker, respectively, are very similar. In contrast, the relative response of the net flow value of the marginal match in the LC model is significantly larger than in the benchmark model.¹⁸ Consequently, one should expect that the LC model generates larger employment fluctuations.

On impact, the elasticity of new job matches in the LC model is about three times larger than in the benchmark model. In the following periods, however, firms' hiring activities decrease sharply. As a result, the impulse response function of employment peaks already after about 9 months. This is due to the modified hiring mechanism. Given linear vacancy posting costs, firms post vacancies in order to equalize expected marginal hiring costs $\kappa/q(\gamma_t)$ and the expected present value of the marginal match.¹⁹ In contrast to the benchmark model, expected marginal hiring costs in the LC model depend on the inverse vacancy filling rate $1/q(\gamma_t)$ and not on the gross hiring rate x_t . In response to the technology shock, the inverse vacancy filling rate increases by about 13 percent and then remains persistently well above its steady state value over the whole observation period. This behavior differs substantially from the rather moderate and temporary increase of the gross hiring rate in the benchmark model.

Since firms are forward looking, they anticipate the future fall in unemployment when deciding upon the optimal number of vacancies. The future fall in unemployment tightens the labor market and, thus, *raises* the expected marginal matching costs in the future. For this reason, firms post vacancies instantaneously as long as the number of unemployed job searchers is still high. This pattern makes it impossible for the LC model to generate a hump-shaped impulse response function of vacancies. Instead, vacancies spike on impact and fall back half way only one period later. This behavior is in sharp contrast to the empirical evidence.

In the benchmark model, however, the mechanism works into the other direction. Firms' expected marginal hiring costs depend on the gross hiring rate x_t . This implies that a high level of employment (i.e. a low level of unemployment) *lowers* expected marginal costs. In comparison to the inverse vacancy filling rate, the gross hiring rate is much less elastic *and* much less persistent. This removes firms' incentive to adjust employment instantaneously. On the contrary, it gives firms strong incentives to smooth hiring over a long period. Hence, the overall employment impact in the LC model is substantially lower than in the benchmark model.

The Interactions between Strategic Wage Bargaining and Convex Labor Adjustment Costs

So far, we have found that (i) strategic wage bargaining amplifies labor

¹⁸Note that the absolute value of the net flow value of the marginal match in the benchmark model is about twice as large than in the LC model (see footnote 12).

¹⁹See Appendix 1.A.1 for the firm's problem in the LC model.

market fluctuations and (ii) convex labor adjustment costs account for hump-shaped impulse response functions. Indeed, beyond understanding how both modifications work in isolation, it is important to explore their interactions.

As discussed above, the impact of current labor market conditions on expected marginal matching costs depends crucially on the specification of firms' hiring costs. In the LC model, a tight labor market *raises* expected marginal matching costs. In the benchmark model, in contrast, a high level of employment *lowers* expected marginal matching costs. Consequently, strategic wage bargaining amplifies the elasticity of labor market variables through two channels. On the one hand, strategic wage bargaining dampens the cyclical fluctuations in the wage bill per worker. This stimulates firms' hiring activities. On the other hand, the higher the stock of employment, the lower the costs of labor adjustment. Thus, the introduction of convex labor adjustment costs increases not only labor market persistence, but also its cyclical fluctuations.

Furthermore, we observe that labor market variables in the benchmark model are even somewhat more persistent than in the NB model. This is due to the fact that strategic wage bargaining removes the impact of the hump-shaped outside alternative. Hence, the net flow value of the marginal match is more persistent, translating into more persistent labor market fluctuations. In summary, we note that strategic wage bargaining and convex labor adjustment costs are complementary in generating elastic *and* persistent labor market responses.

Robustness of the Hump-Shaped Vacancy Responses In the following, we examine whether the hump-shaped impulse response function of vacancies in the benchmark model is robust with respect to the two distortionary effects – strategic wage bargaining and the value of firms' bargaining power.

As shown in a previous paragraph, vacancies in the NB model are much less elastic than in the benchmark model. In addition, the hump-shape is flattened considerably.

Therefore, we evaluate the benchmark model with firms' bargaining power set to unity. On impact, vacancies rise by about 15 percent. This increase is about one and a half times higher than in the case of symmetrically distributed bargaining power. Moreover, vacancies display a marked hump-shape, albeit the hump is slightly weaker than in the benchmark model.

However, social optimality under convex labor adjustment costs requires that the wage bill per worker equals household's outside alternative. This condition is only satisfied if we assume Nash bargaining *and* if we set firms' bargaining power equal to unity. We now observe that vacancies increase on impact by about 7 percent, reach a maximum with about 3 quarters delay, and then return relatively quickly to their steady state value. These results indicate that the combination of both distortionary effects dampens the hump to some degree. Nevertheless, the hump-shaped pattern of vacancies is a robust

result of our benchmark model.

Simulation Results

This section evaluates the benchmark model against U.S. data. Thereby, our analysis focuses on the cyclical behavior of vacancies and unemployment (table 1.5). In particular, we examine the model in terms of its capability to generate sufficient volatility *and* persistence in both variables.

The Benchmark Model Strategic wage bargaining makes the wage bill per worker independent of fluctuations in household's outside alternative. Hence, the wage bill per worker (wl) is significantly less volatile than output per worker (y/n), giving firms strong incentives to expand their hiring activities in economic upswings. Thus, the benchmark model replicates closely the cyclical volatility of vacancies (v), unemployment ($1 - n$) and market tightness (γ). This result is in line with the insight in Hall and Milgrom (2008): Strategic wage bargaining generates endogenous real wage rigidity. This increases the volatility of the net flow value of the marginal match. As a result, labor market variables become more volatile.

Furthermore, we note that vacancies, unemployment and market tightness are highly persistent. This can be ascribed to the modified hiring condition which alters the qualitative pattern of firms' hiring behavior. Consequently, the benchmark model generates hump-shaped responses in unemployment *and* vacancies. For the same reason, the benchmark model is capable to replicate the U-shaped pattern of the dynamic Beveridge curve (see table 1.6 and figure 1.3). Consistent with the data, the negative relation between model generated vacancies and unemployment remains for more than 4 quarters.

Apart from that, the benchmark model accounts for the fact that unemployment and market tightness lag the cycle by one quarter. This indicates that the combination of strategic wage bargaining and convex labor adjustment costs enhances the model's ability to propagate technology shocks in the labor market. On the other hand, the benchmark model cannot match the cyclical co-movement of two other variables – output per worker and the wage bill per worker. In the data, the contemporaneous correlation between output and output per worker is close to unity. The wage bill per worker, in contrast, shows a much weaker contemporaneous correlation with output. Table 1.7 displays that both variables are only moderately positively correlated. In the model, however, we observe that output per worker and the wage bill per worker are perfectly correlated.

Indeed, the almost perfect correlation between output per worker and the wage bill per worker is generated essentially by construction. Equation (1.31) shows that variations in the wage bill per worker are closely related to changes in output per worker. Since this paper is motivated by the cyclical behavior of vacancies and unemployment, we allow only for total factor productivity shocks. Yet, we conjecture that adding a shock to the value

of bargaining may help to bring the co-movement of labor market variables closer to the data.

The Impact of Strategic Wage Bargaining In the NB model, both parties receive period-by-period a constant share of the mutual surplus. For this reason, the wage bill per worker is almost as volatile as output per worker, giving firms little incentive to adjust employment over the business cycle. This contrasts sharply with the data. Consequently, the cyclical fluctuations of vacancies and unemployment are insufficiently small. The same applies to market tightness, confirming the conclusion reached by Shimer (2005).

On the other hand, the Nash bargaining assumption does not alter the *qualitative* pattern of employment adjustment. The model generated time series of vacancies and unemployment remain almost as persistent as in the benchmark model. As a result, the dynamic Beveridge curve maintains the U-shaped pattern. Even though, we note that the negative relation between vacancies and unemployment remains now only for somewhat more than 3 quarters (instead of more than 4 quarters in the benchmark model). This might be due to the fact that Nash bargaining reduces not only the volatility, but also the persistence of the net flow value of the marginal match.

The Impact of Convex Labor Adjustment Costs Due to strategic wage bargaining, we observe that the wage bill per worker is about half as volatile as output per worker, giving firms strong incentives to amplify hiring activities. This result holds independently of the hiring cost function. In the LC model, however, we observe that vacancies spike on impact and fall back very quickly. Consequently, the cumulative inflow of new job matches in the LC model is much weaker than in the benchmark model, inducing less volatility in employment, unemployment and market tightness.

For the same reason, all labor market variables are much less persistent. This pattern can be ascribed to the modified hiring condition. Given linear vacancy posting costs, firms anticipate the fall in the vacancy filling ratio and, hence, adjust employment instantaneously. On the contrary, convex labor adjustment costs give firms strong incentives to smooth their hiring activities over several periods. This causes the continuous inflow of new job matches in the benchmark model, generating highly persistent labor market variables. Thus, as pointed out by Yashiv (2006), convex labor adjustment costs improve the performance of the job search and matching model considerably.

In particular, the first order autocorrelation of vacancies in the LC model is much weaker than in the benchmark model. This follows directly from the counter-factual shape of the impulse response function under linear vacancy posting costs. Moreover, the shape of the dynamic Beveridge curve is biased. Despite the strong negative contemporaneous correlation, the cross-correlation between unemployment and leaded vacancies is close to zero beyond 2 quarters. In other words, the LC model predicts rather a J-shaped

relationship, echoing the findings of Fujita (2004).

The Interactions between Strategic Wage Bargaining and Convex Labor Adjustment Costs We summarize that (i) strategic wage bargaining amplifies the volatility of the labor market and (ii) convex labor adjustment costs improve labor market persistence. Therefore, we conclude that only the combination of both features generates sufficient volatility and persistence in the labor market.

Furthermore, the results presented above indicate that strategic wage bargaining and convex labor adjustment costs are complementary in generating volatility and persistence. This interesting result stems from the specification of the hiring cost function. Following Gertler and Trigari (2009), firms' hiring costs now depend negatively on the employment level. Hence, large labor market fluctuations dampen the cyclical variations of firm's hiring costs. For this reason, strategic wage bargaining amplifies the volatility of labor market variables through two channels. On the one hand, strategic wage bargaining amplifies firms' hiring activities in economic upswings. On the other hand, the higher the stock of employment, the lower the costs of labor adjustment. Consequently, the introduction of convex labor adjustment costs enhances the cyclical volatility of unemployment and market tightness. The volatility of vacancies, however, remains virtually unchanged.

In addition, strategic wage bargaining removes the impact of the hump-shaped outside alternative on the wage bill per worker. Thus, strategic wage bargaining induces not only more labor market volatility, but also more labor market persistence.

The complementarity between strategic wage bargaining and convex labor adjustment costs is also illustrated by the dynamic Beveridge curve. Only if we combine both features, the negative relation between vacancies and unemployment remains for more than 4 quarters. Clearly, the impact of convex labor adjustment costs seems to be more important in this respect.

Business Cycle Analysis The last section has shown that the benchmark model replicates the cyclical behavior of the labor market remarkably well. In the following, we analyze the business cycle properties of the benchmark model more comprehensively. The main features of the US business cycle are well-known (Cooley and Prescott, 1995): The fluctuations of output y and total hours nl are nearly equal, while consumption c fluctuates less and investment i fluctuates more. Employment n is almost as volatile as output, indicating that fluctuations in total hours are generated for the most part by the extensive margin. This conjecture is confirmed by the relatively tiny fluctuations in hours per worker l . In addition, also labor productivity $y/(nl)$ and the real wage rate w fluctuate considerably less than output. All these variables are pro-cyclical, albeit labor productivity and real wages show clearly less pro-cyclical variations than the other variables.

Table 1.8 compares the business cycle statistics of the benchmark model with the data. In total, the benchmark model captures properly the cyclical behavior of consumption, investment and employment. In particular, the benchmark model works well along the extensive margin of cyclical employment adjustment. Beyond matching the standard business cycle facts, the benchmark model accounts additionally for the low and positive correlation between employment and the wage bill per worker found in U.S. data (see table 1.9).

Furthermore, the data reveal that the empirical correlation between total hours and the real wage rate is essentially zero. This pattern is often referred to as the “Dunlop-Tarshis observation”.²⁰ However, we observe that the benchmark model cannot match this stylized fact as closely as the other features of the U.S. business cycle. Nevertheless, the benchmark model performs much better than the alternative model specifications. The failure to match the Dunlop-Tarshis observation indicates that the model cannot replicate the cyclical co-movement of hours per worker.²¹ In the data, hours per worker are pro-cyclical. In the model, the contemporaneous correlation between output and hours per worker is close to zero. This artifact follows from the interactions between strategic wage bargaining and convex labor adjustment costs. As described above, the combination of both modifications induces larger employment fluctuations than the other model specifications. This leads to larger output fluctuations, implying a strong income effect. On the other hand, the combination of strategic wage bargaining and convex labor adjustment leads to a fast decline in labor productivity. This implies that the intertemporal substitution effect is relatively weak. Consequently, workers grant more value to leisure and, thus, make less (additional) labor effort in economic upswings.

Apart from that, the counter-factual behavior of hours per worker biases also the cyclical properties of some other variables, like the real wage rate. Given that the real wage rate is defined as the wage bill per worker over hours per worker, the real wage rate has to account for almost the whole pro-cyclical of the individual wage bill. As a result, the model generated real wage rate is highly pro-cyclical – in stark contrast to the data. Furthermore, labor productivity is too pro-cyclical and total hours are too less volatile.

For the same reason, we observe that the benchmark model cannot fully account for the relatively high volatility of the aggregate wage bill. In addition, the aggregate wage bill is too pro-cyclical. Hence, the benchmark model generates too much volatility in the labor share and underestimates its lead. Yet, the benchmark model still improves the dynamic behavior of the labor share slightly compared to previous studies (Andolfatto, 1996).

Moreover, we note that output volatility is slightly lower than in the data. This

²⁰See, *inter alia*, Christiano and Eichenbaum (1992) and the references therein.

²¹Note that we observe hours per worker in the data. In the model, however, l_t might capture rather (unobservable) labor effort.

may be due to the somewhat understated volatility in total hours and investment. However, it is likely to increase output volatility by allowing for variable capital utilization (Burnside and Eichenbaum, 1996).

Finally, we analyze the cyclical behavior of the wage bill per worker, relative to the cyclical behavior of the bargaining set. As explained above, the wage bill per worker satisfies individual rationality only if it lies in the bargaining set. For this purpose, figure 1.4 reports the evolution of the reservation value of the firm (upper graph), the wage bill per worker (middle graph) and the reservation value of the household (lower graph) over 12000 simulated periods. We highlight the steady state value of household's reservation value as well as its 95% confidence interval. The graphs show that the wage bill per worker is always in the bargaining set during any period in this long simulation. Moreover, the upper confidence bound of household's reservation value is far below the graph of the wage bill per worker. Consequently, all employment formations are efficient (Hall, 2005a). In other words, the critique of Barro (1977) does not apply here.

1.4 Conclusion

This paper modifies the standard Mortensen-Pissarides job matching model in order to explain the cyclical behavior of vacancies and unemployment. The modifications include convex labor adjustment costs and strategic wage bargaining as introduced into the literature by Hall and Milgrom (2008). The main contribution of our paper is to improve the cyclical behavior of vacancies and unemployment along two dimensions – volatility and persistence.

First, we show that strategic wage bargaining increases the volatility of both variables enormously. This is due to the fact that strategic wage bargaining makes the wage bill per worker independent of the fluctuations in household's outside alternative. As a result, the elasticity of firms' costs per worker is reduced by half. Hence, firms have much stronger incentives to hire new workers in economic upswings.

Second, the introduction of convex labor adjustment costs leads to more persistent labor market responses. In particular, the impulse response function of vacancies shows a marked hump-shape, peaking with several quarters delay. This can be attributed to the firms' altered optimization problem. In contrast to the case of linear vacancy posting costs, firms' hiring costs now depend on the number of vacancies that are *filled*, and not on the number of vacancies that are *posted*. Thus, marginal hiring costs under convex labor adjustment costs are less volatile and less persistent than under linear vacancy posting costs, giving firms strong incentives to smooth their hiring activities.

Moreover, we observe that strategic wage bargaining and convex labor adjustment costs are complementary in generating labor market volatility *and* persistence. This interesting result stems from the specification of the hiring cost function. Following

Gertler and Trigari (2009), we assume that firms' hiring costs depend negatively on the employment level. For this reason, strategic wage bargaining amplifies the elasticity of labor market variables through two channels. On the one hand, strategic wage bargaining enhances employment volatility. On the other hand, large labor market fluctuations dampen the cyclical variations of firms' hiring costs. Consequently, the introduction of convex labor adjustment costs induces not only more persistence, but also more volatility in the labor market.

Apart from that, we find that our model gives rise to two distortionary effects. Given convex labor adjustment costs, social optimality requires that the wage bill per worker is equal to the household's outside alternative. In contrast, we assume that (i) the wage bill per worker is independent of the fluctuations in household's outside alternative and (ii) firms' bargaining power ξ is strictly smaller than unity. Therefore, firms' private gains from search effort are generally smaller than their social contribution. In this case, the dynamic behavior of the wage bill per worker is not socially optimal (Hosios, 1990).

It would be interesting to extend our analysis toward endogenizing the value of bargaining. To our knowledge, the only paper that attempts to address this issue is by Knabe (2005). The study of such questions, however, is beyond the scope of this paper.

Appendix 1.A Computations

1.A.1 Firm's Problem with Linear Vacancy Posting Costs

Corresponding equations (1.13) - (1.15), the optimization problem of the representative firm is:

$$\mathcal{V}(\Omega_t^F) = \max_{k_t, v_t} \left\{ y_t - w_t n_t l_t - r_t k_t - \kappa v_t + \beta E_t \left[(\lambda_{t+1}/\lambda_t) \mathcal{V}(\Omega_{t+1}^F) \right] \right\}, \quad (1.33)$$

s.t.

$$y_t = a_t k_t^\theta (n_t l_t)^{(1-\theta)}, \quad (1.34)$$

$$n_{t+1} = (1 - \sigma)n_t + q(\gamma_t)v_t. \quad (1.35)$$

Corresponding equations (1.16) - (1.17), the first order conditions are given as:

$$k_t : r_t = \theta \frac{y_t}{k_t}, \quad (1.36)$$

$$n_{t+1} : \frac{\kappa}{q(\gamma_t)} = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left((1 - \theta) \frac{y_{t+1}}{n_{t+1}} - w_{t+1} l_{t+1} + (1 - \sigma) \frac{\kappa}{q(\gamma_{t+1})} \right) \right]. \quad (1.37)$$

Corresponding equation (1.18), the resource constraint of the economy is:

$$y_t = c_t + k_{t+1} - (1 - \delta)k_t + \kappa v_t. \quad (1.38)$$

Corresponding equation (1.30) the wage bill per worker under Nash bargaining is:

$$w_t l_t = (1 - \xi) \left[(1 - \theta) \frac{y_t}{n_t} \right] + \xi \left[\frac{u_t^U - u_t^N}{\lambda_t} + \frac{(1 - \xi)}{\xi} \kappa \frac{v_t}{(1 - n_t)} \right]. \quad (1.39)$$

Corresponding equation (1.31) the wage bill per worker under strategic wage bargaining is:

$$w_t l_t = (1 - \xi) \left[(1 - \theta) \frac{y_t}{n_t} \right] + \xi \left[\frac{\bar{u}^U - \bar{u}^N}{\bar{\lambda}} + \frac{(1 - \xi)}{\xi} \kappa \frac{\bar{v}}{(1 - \bar{n})} \right]. \quad (1.40)$$

Hence, we replace condition (1.17) by (1.37), condition (1.18) by (1.38), condition (1.30) by (1.39), and condition (1.31) by (1.40) in order to obtain to competitive equilibrium.

1.A.2 Social Planner Solution

The set $\mathcal{U}(\Omega_t) = \{k_t, n_t\}$ denotes the state space of the social planner.

$$\begin{aligned} \mathcal{U}(\Omega_t) &= \max_{c_t, l_t, k_{t+1}, n_{t+1}, v_t} \\ &\left\{ \ln(c_t) + n_t \phi_1 \frac{(1 - l_t)^{1-\eta}}{1 - \eta} + (1 - n_t) \phi_2 \frac{(1 - e)^{1-\eta}}{1 - \eta} + \beta E_t [\mathcal{U}(\Omega_{t+1})] \right\}, \end{aligned} \quad (1.41)$$

s.t.

$$k_{t+1} = F(k_t, n_t l_t) + (1 - \delta)k_t - \frac{\kappa}{2} x_t^2 n_t - c_t, \quad (1.42)$$

$$n_{t+1} = (1 - \sigma)n_t + m_t. \quad (1.43)$$

The first order conditions are given as:

$$c_t : \lambda_t = 1/c_t, \quad (1.44)$$

$$l_t : \lambda_t F_2(k_t, n_t l_t) = \phi_1 (1 - l_t)^{-\eta}, \quad (1.45)$$

$$k_{t+1} : \lambda_t = \beta E_t [\mathcal{U}_1(\Omega_{t+1})], \quad (1.46)$$

$$n_{t+1} : \mu_t = \beta E_t [\mathcal{U}_2(\Omega_{t+1})], \quad (1.47)$$

$$v_t : \mu_t = \lambda_t \kappa x_t. \quad (1.48)$$

The envelope conditions are given as:

$$\mathcal{U}_1(\Omega_t) = \lambda_t [F_1(k_t, n_t l_t) + 1 - \delta], \quad (1.49)$$

$$\begin{aligned} \mathcal{U}_2(\Omega_t) = & \phi_1 \frac{(1 - l_t)^{1-\eta}}{1 - \eta} - \phi_2 \frac{(1 - e)^{1-\eta}}{1 - \eta} + \\ & \lambda_t F_2(k_t, n_t l_t) l_t + \mu_t \left[1 - \sigma - (1 - \alpha) \frac{m_t}{1 - n_t} \right]. \end{aligned} \quad (1.50)$$

Consequently, the social planner solution is defined by following conditions:

$$\lambda_t = \beta E_t \lambda_{t+1} [F_1(k_{t+1}, n_{t+1} l_{t+1}) + 1 - \delta], \quad (1.51)$$

$$\lambda_t F_2(k_t, n_t l_t) = \phi_1 (1 - l_t)^{-\eta}, \quad (1.52)$$

$$\mu_t = \lambda_t \kappa x_t, \quad (1.53)$$

$$\begin{aligned} \mu_t = & \beta E_t \left[\phi_1 \frac{(1 - l_{t+1})^{1-\eta}}{1 - \eta} - \phi_2 \frac{(1 - e)^{1-\eta}}{1 - \eta} + \right. \\ & \left. \lambda_{t+1} F_2(k_{t+1}, n_{t+1} l_{t+1}) l_{t+1} + \mu_{t+1} \left[1 - \sigma - (1 - \alpha) \frac{m_{t+1}}{1 - n_{t+1}} \right] \right] \end{aligned} \quad (1.54)$$

$$k_{t+1} = F(k_t, n_t l_t) + (1 - \delta) k_t - \frac{\kappa}{2} \frac{m_t^2}{n_t} - c_t, \quad (1.55)$$

$$n_{t+1} = (1 - \sigma) n_t + m_t. \quad (1.56)$$

1.A.3 The Market Solution is generally not Socially Optimal

The surplus S_t (in utility units) equals the social benefit the marginal match:

$$\lambda_t S_t = \mathcal{U}_2(\Omega_t). \quad (1.57)$$

We substitute this result into the first order condition (1.47):

$$\mu_t = \beta E_t [\lambda_{t+1} S_{t+1}]. \quad (1.58)$$

The Nash sharing rule (1.28) implies that the firm gains the share ξ of the surplus:

$$\mathcal{V}_1(\Omega_t^F) = \xi S_t. \quad (1.59)$$

Hence:

$$\mu_t = \beta \xi^{-1} E_t [\lambda_{t+1} \mathcal{V}_1(\Omega_{t+1}^F)]. \quad (1.60)$$

Recall the first order condition of the firm (1.17):

$$\kappa x_t = \beta E_t \left[(\lambda_{t+1}/\lambda_t) \mathcal{V}_1(\Omega_{t+1}^F) \right] \quad (1.61)$$

Substituting (1.60) into (1.61) yields:

$$\kappa x_t = \beta \mu_t \xi E_t \left[(\lambda_{t+1}/\lambda_t) \beta^{-1} \lambda_{t+1}^{-1} \right] \quad (1.62)$$

Hence:

$$\kappa x_t \lambda_t = \mu_t \xi \quad (1.63)$$

Instead, the first order condition of the social planner (1.48) postulates:

$$\lambda_t \kappa x_t = \mu_t. \quad (1.64)$$

Hence, the market solution is socially optimal, if and only if $\xi = 1$ holds.

Appendix 1.B Tables

Table 1.1: The Monthly Parameterization of the Model

Description	Variable	Value	Source
Technology			
production elasticity of capital	θ	0.40	Cooley and Prescott (1995)
depreciation rate	δ	0.0083	Kydland and Prescott (1982)
Preferences			
discount factor	β	0.9967	Kydland and Prescott (1982)
working time per worker	\bar{l}	1/3	Juster and Stafford (1991)
effort per job seeker	e	1/12	Barron and Gilley (1981)
individual labor supply elasticity	ν	0.5	MaCurdy (1983)
Labor Market			
job destruction rate	σ	0.035	Shimer (2007)
unemployment rate	$1 - \bar{n}$	0.10	Hall (2005b)
vacancy filling rate	$q(\bar{\gamma})$	0.3381	van Ours and Ridder (1992)
adjustment costs/ output ratio	ψ	0.01	Hamermesh and Pfann (1996)
matching elasticity of vacancies	α	0.5	Petrongolo and Pissarides (2001)
firm's bargaining power	ξ	0.5	Svejnar (1986)
Technology Shock			
1st order autocorrelation	ρ	0.9830	Cooley and Prescott (1995)
standard deviation	σ_ε	0.004395	Cooley and Prescott (1995)

Table 1.2: Implied Steady State Values

Description	Variable	Value	Description	Variable	Value
job finding rate	$q(\bar{\gamma})\bar{\gamma}$	0.3150	vacancies	\bar{v}	0.0932
matches	\bar{m}	0.0315	gross hiring rate	\bar{x}	0.0350
hiring costs parameter	κ	18.1406	matching function parameter	χ	1.1305
real interest rate	\bar{r}	0.0034	capital	\bar{k}	34.2200
investment	$\delta\bar{k}$	0.2852	consumption	\bar{c}	0.7048
aggregate wage bill/labor share	$\bar{n}\bar{w}\bar{l}$	0.5881	wage bill per worker	$\bar{w}\bar{l}$	0.6534
production function parameter	ζ	0.5012	real wage	\bar{w}	1.9603
household's reservation value	$(\bar{w}l)_{min}$	0.0164	firm's reservation value	$(\bar{w}l)_{max}$	1.2905
leisure parameter employed	ϕ_1	0.5605	leisure parameter unemployed	ϕ_2	0.0504
leisure exponent	η	4	replacement ratio	\bar{b}	0.6331

Table 1.3: Raw Data Series

Key	Raw Series	Frequency	Database	Series ID
[1]	Labor Force	monthly	St. Louis Fed: FRED®	CLF16OV
[2]	Unemployment	monthly	http://www.stanford.edu/~rehall/MA-7-13-05.xls	JF rate calcs (G195:G626)
[3]	Vacancies	monthly	St. Louis Fed: FRED®	HELPWANT
[4]	Hours per Worker	monthly	St. Louis Fed: FRED®	AWHNONAG
[5]	Total Hours	monthly	St. Louis Fed: FRED®	AWHI
[6]	Real Wage	quarterly	St. Louis Fed: FRED®	COMPRNFB
[7]	Durable Goods	quarterly	St. Louis Fed: FRED®	PCDGC96
[8]	Nondurable Goods	quarterly	St. Louis Fed: FRED®	PCNDGC96
[9]	Services	quarterly	St. Louis Fed: FRED®	PCESVC96
[10]	Investment	quarterly	St. Louis Fed: FRED®	GPDIC96

Table 1.4: Constructed Data Series

Key	Constructed Series	Variable	Construction
(1)	Consumption	c	$= ([8] + [9])/[1]$
(2)	Investment	i	$= ([7] + [10])/[1]$
(3)	Output	y	$= (1) + (2)$
(4)	Employment	n	$= ([1] - [2])/[1]$
(5)	Unemployment	$1 - n$	$= [2]/[1]$
(6)	Vacancies	v	$= [3]/[1]$
(7)	Market Tightness	$v/(1 - n)$	$= (6)/(5)$
(8)	Hours per Worker	l	$= [4]$
(9)	Total Hours	$n \cdot l$	$= [5]$
(10)	Real Wage	w	$= [6]$
(11)	Aggregate Wage Bill	$w \cdot n \cdot l$	$= (9) \cdot (10)$
(12)	Labor's Share	$(w \cdot n \cdot l)/y$	$= (11)/(3)$
(13)	Labor Productivity	$y/(n \cdot l)$	$= (3)/(9)$
(14)	Output per Worker	y/n	$= (3)/(4)$
(15)	Individual Wage Bill	$w \cdot l$	$= (10) \cdot (8)$

Table 1.5: Simulation Results. This table shows the results of the model simulations. For each variable, we report the relative standard deviation (σ_X/σ_Y), the first order autocorrelation ($\rho_{X_T, X_{T+1}}$), the phase shift relative to output (in parenthesis), and the contemporaneous correlation with output (ρ_{XY}).

	v	$1-n$	γ	y/n	wl
U.S. Business Cycle Facts					
σ_X/σ_Y	7.36	6.29	13.49	0.83	0.60
$\rho_{X_T, X_{T+1}}$	0.92	0.90	0.92	0.85	0.86
ρ_{XY}	(+1) 0.85	(+1) -0.79	(+1) 0.83	(0) 0.98	(0) 0.55
Strategic Wage Bargaining with Convex Labor Adjustment Costs					
σ_X/σ_Y	9.84	7.08	16.24	0.63	0.37
$\rho_{X_T, X_{T+1}}$	0.87	0.93	0.91	0.68	0.68
ρ_{XY}	(0) 0.99	(+1) -0.90	(+1) 0.94	(-1) 0.62	(-1) 0.62
Nash Bargaining with Convex Labor Adjustment Costs					
σ_X/σ_Y	1.42	1.00	2.30	0.92	0.88
$\rho_{X_T, X_{T+1}}$	0.83	0.91	0.89	0.76	0.77
ρ_{XY}	(0) 0.99	(+1) -0.88	(0) 0.93	(0) 1.00	(0) 1.00
Strategic Wage Bargaining with Linear Vacancy Posting Costs					
σ_X/σ_Y	10.01	6.19	14.82	0.42	0.22
$\rho_{X_T, X_{T+1}}$	0.60	0.83	0.76	0.57	0.57
ρ_{XY}	(0) 0.88	(0) -0.99	(0) 0.98	(-1) 0.85	(-1) 0.85

Table 1.6: The Dynamic Beveridge Curve. The table shows the correlation coefficients between unemployment u_t and vacancies v_{t+k} , lagged respectively leaded by k quarters.

U.S. Data	-0.13	-0.34	-0.56	-0.75	-0.90	-0.95	-0.86	-0.67	-0.45	-0.20	0.03
Benchmark Model	-0.30	-0.50	-0.69	-0.86	-0.96	-0.94	-0.76	-0.54	-0.32	-0.13	0.04
NB Model	-0.22	-0.41	-0.62	-0.82	-0.95	-0.93	-0.71	-0.46	-0.23	-0.04	0.11
LC Model	-0.07	-0.22	-0.40	-0.62	-0.84	-0.90	-0.51	-0.22	-0.05	0.06	0.14
	-5	-4	-3	-2	-1	0	1	2	3	4	5

Table 1.7: The Dynamic Cross-Correlation Pattern. The table shows the correlation coefficients between the wage bill per worker $w_t l_t$ and output per worker y_{t+k}/n_{t+k} , lagged respectively leaded by k quarters.

U.S. Data	-0.12	0.00	0.14	0.26	0.40	0.53	0.53	0.46	0.38	0.27	0.14
SB Model	-0.17	-0.07	0.09	0.33	0.68	1.00	0.68	0.33	0.09	-0.06	-0.16
	-5	-4	-3	-2	-1	0	1	2	3	4	5

Table 1.8: Business Cycle Statistics. For each variable, the table reports the relative standard deviation (σ_X/σ_Y), the first order autocorrelation ($\rho_{X_T, X_{T+1}}$), the phase shift relative to output (in parenthesis), and the contemporaneous correlation with output (ρ_{XY}).

	c	i	nl	n	l	w	$y/(nl)$	nwl	$(nl)/y$
U.S. Business Cycle Facts									
$\sigma_Y = 1.78$									
σ_X/σ_Y	0.42	3.37	1.06	0.87	0.24	0.54	0.71	1.20	0.74
$\rho_{X_T, X_{T+1}}$	0.75	0.83	0.91	0.90	0.74	0.84	0.84	0.91	0.85
ρ_{XY}	(0) 0.81	(0) 0.97	(+1) 0.76	(+1) 0.82	(0) 0.77	(-1) 0.27	(-4) 0.27	(+1) 0.79	(-5) -0.07
Benchmark Model									
$\sigma_Y = 1.32$									
σ_X/σ_Y	0.42	2.37	0.68	0.79	0.17	0.26	0.63	0.86	0.26
$\rho_{X_T, X_{T+1}}$	0.81	0.87	0.93	0.93	0.83	0.69	0.67	0.90	0.68
ρ_{XY}	(0) 0.97	(0) 0.99	(+1) 0.88	(+1) 0.90	(+4) -0.03	(0) 0.88	(0) 0.78	(0) 0.97	(-1) -0.63

Table 1.9: The Dunlop-Tarshis observation. The table reports the correlation coefficients between employment n_t and the wage bill per worker $w_t l_t$ as well as the correlation coefficients between total hours $n_t l_t$ and the real wage rate w_t , respectively.

	U.S. Data	B'mark Model	NB Model	LC Model
$\rho(n_t, w_t l_t)$	0.34	0.22	0.87	0.87
$\rho(n_t l_t, w_t)$	0.03	0.55	0.98	0.88

Appendix 1.C Figures

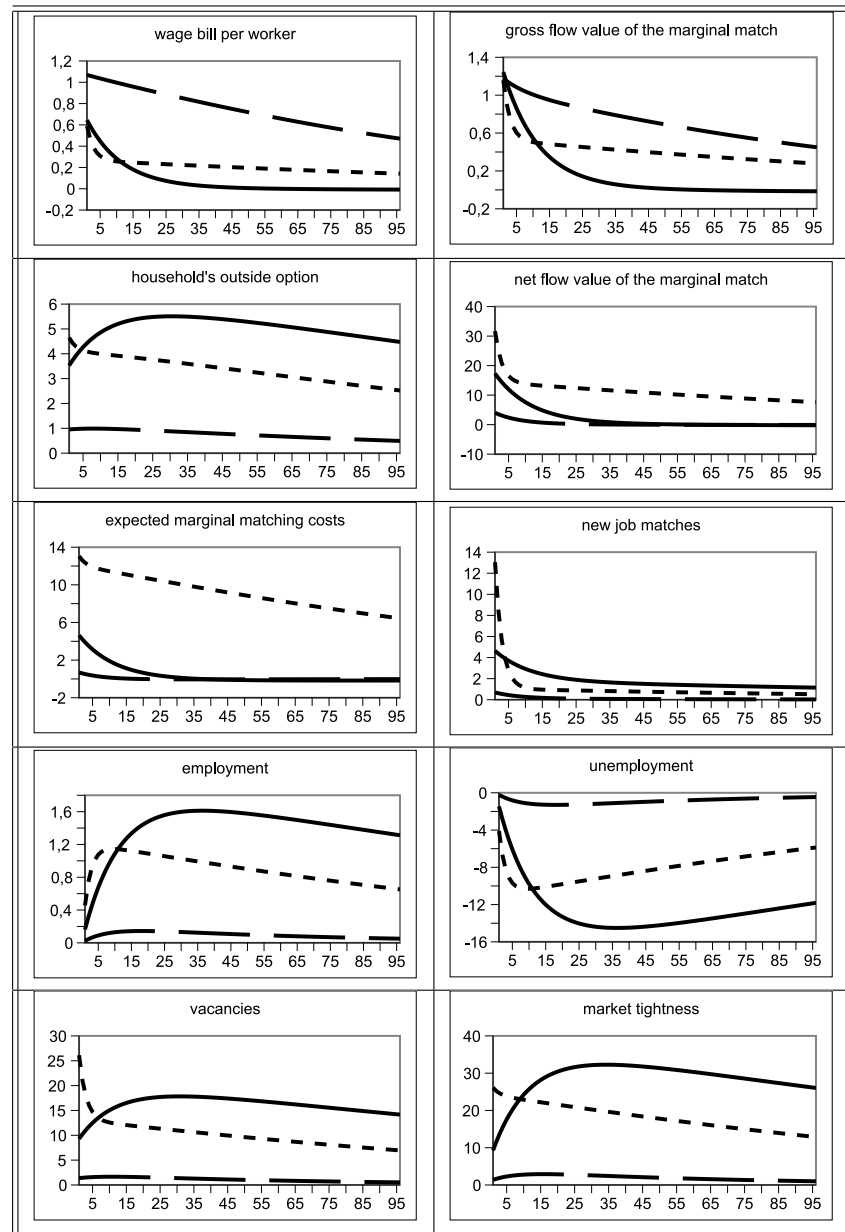


Figure 1.1: Impulse Response Functions. The graphs depict the evolution of the benchmark model (solid line), the NB model (dashed line), and the LC model (dotted line) over 96 months (32 quarters).

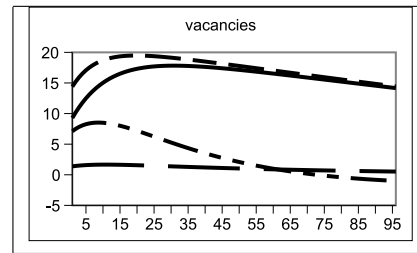


Figure 1.2: Robust Hump-Shaped Vacancy Dynamics. The solid line represents the benchmark model. The dashed line represents the NB model. The dot-dashed line represents the benchmark model with firms' bargaining power equal to unity. The dot-dot-dashed line represents the NB model with firms' bargaining power equal to unity.

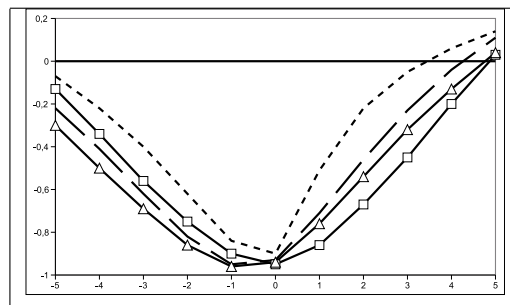


Figure 1.3: The Dynamic Beveridge Curve (graphical representation of table 1.6). The solid line with square shaped markers represents U.S. data. The solid line with triangle-shaped markers represents the benchmark model. The dashed line represents the NB model. The dotted line represents the LC model.

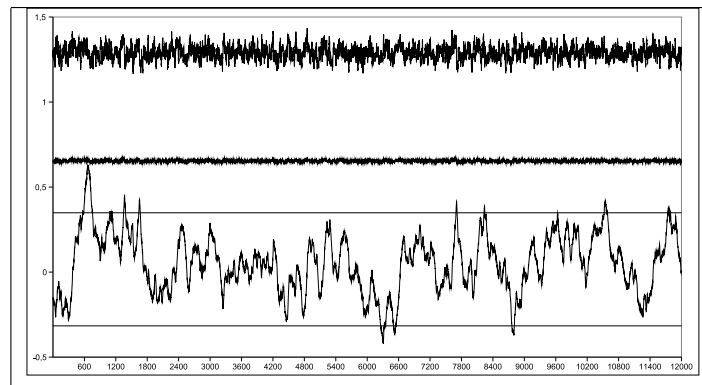


Figure 1.4: The Bargaining Set. The graphs depict the evolution of the reservation value of the firm (upper graph), the wage bill per worker (middle graph) and the reservation value of the worker (lower graph) over 12000 simulated periods. In addition, we highlight the steady state value of the worker's reservation value as well as its 95% confidence interval.

Bibliography

- Andolfatto, D. (1996), ‘Business cycles and labor-market search’, *American Economic Review* **86**(1), 112–132.
- Barro, R. (1977), ‘Long-term contracting, sticky prices, and monetary policy’, *Journal of Monetary Economics* **3**(3), 305–316.
- Barron, J. M. and Gilley, O. W. (1981), ‘Job search and vacancy contacts: Note’, *American Economic Review* **71**(4), 747–52.
- Binmore, K., Rubinstein, A. and Wolinsky, A. (1986), ‘The Nash bargaining solution in economic modeling’, *The RAND Journal of Economics* **17**(2), 176–188.
- Blanchard, O. J., Diamond, P., Hall, R. E. and Yellen, J. (1989), ‘The Beveridge curve’, *Brookings Papers on Economic Activity* **1**, 1–76.
- Blundell, R. and Macurdy, T. (1999), Labor supply: A review of alternative approaches, in O. Ashenfelter and D. Card, eds, ‘Handbook of Labor Economics’, Vol. 3 of *Handbook of Labor Economics*, Elsevier, chapter 27, pp. 1559–1695.
- Blundell, R., Meghir, C., Symons, E. and Walker, I. (1988), ‘Labour supply specification and the evaluation of tax reforms’, *Journal of Public Economics* **Volume 36**(Issue 1), Pages 23–52.
- Braun, H., Bock, R. D. and DiCecio, R. (2006), Aggregate shocks and labor market fluctuations, Working Papers No. 2006-004, Federal Reserve Bank of St. Louis.
- Browning, M., Hansen, L. P. and Heckman, J. J. (1999), Micro data and general equilibrium models, in J. B. Taylor and M. Woodford, eds, ‘Handbook of Macroeconomics’, Vol. 1 of *Handbook of Macroeconomics*, Elsevier, chapter 8, pp. 543–633.
- Burnside, C. and Eichenbaum, M. (1996), ‘Factor-hoarding and the propagation of business-cycle shocks’, *American Economic Review* **86**(5), 1154–1174.
- Cheron, A. and Langot, F. (2004), ‘Labor market search and real business cycles: Reconciling Nash bargaining with the real wage dynamics’, *Review of Economic Dynamics* **7**(2), 476–493.
- Christiano, L. J. and Eichenbaum, M. (1992), ‘Current real-business-cycle theories and aggregate labor-market fluctuations’, *American Economic Review* **82**(3), 430–450.
- Cooley, T. F. and Prescott, E. C. (1995), Economic growth and business cycles, in T. F. Cooley, ed., ‘Frontiers of Business Cycle Research’, Princeton University Press, Princeton, New Jersey, pp. 1–38.
- Costain, J. S. and Reiter, M. (2008), ‘Business cycles, unemployment insurance, and the calibration of matching models’, *Journal of Economic Dynamics and Control* **32**(4), 1120–1155.
- den Haan, W. J., Ramey, G. and Watson, J. (2000), ‘Job destruction and propagation of shocks’, *American Economic Review* **90**(3), 482–498.

- Fujita, S. (2004), Vacancy persistence, Working Papers No. 04-23, Federal Reserve Bank of Philadelphia.
- Fujita, S. and Ramey, G. (2006), The dynamic Beveridge curve, 2006 Meeting Papers No. 239, Society for Economic Dynamics.
- Fujita, S. and Ramey, G. (2007), ‘Job matching and propagation’, *Journal of Economic Dynamics and Control* **31**(11), 3671–3698.
- Gertler, M. and Trigari, A. (2009), ‘Unemployment fluctuations with staggered Nash wage bargaining’, *Journal of Political Economy* **117**(1), 38–86.
- Hagedorn, M. and Manovskii, I. (2008), ‘The cyclical behavior of equilibrium unemployment and vacancies revisited’, *American Economic Review* **98**(4), 1692–1706.
- Hall, R. (2005a), ‘Employment fluctuations with equilibrium wage stickiness’, *American Economic Review* **95**(1), 50–65.
- Hall, R. E. (2005b), Job loss, job finding, and unemployment in the U.S. economy over the past fifty years, NBER Working Paper Series No. 11678, National Bureau of Economic Research.
- Hall, R. E. and Milgrom, P. R. (2008), ‘The limited influence of unemployment on the wage bargain’, *American Economic Review* **98**(4), 1653–1174.
- Hamermesh, D. S. and Pfann, G. A. (1996), ‘Adjustment costs in factor demand’, *Journal of Economic Literature* **34**(3), 1264–1292.
- Hodrick, R. J. and Prescott, E. C. (1997), ‘Postwar U.S. business cycles: An empirical investigation’, *Journal of Money, Credit and Banking* **29**(1), 1–16.
- Holmlund, B. (1998), ‘Unemployment insurance in theory and practice’, *Scandinavian Journal of Economics* **100**(1), 113–141.
- Hornstein, A., Krusell, P. and Violante, G. L. (2005), ‘Unemployment and vacancy fluctuations in the matching model: Inspecting the mechanism’, *Economic Quarterly* **91**(3), 19–50.
- Hosios, A. J. (1990), ‘On the efficiency of matching and related models of search and unemployment’, *Review of Economic Studies* **57**(2), 279–298.
- Jones, S. R. G. and Riddell, W. C. (1999), ‘The measurement of unemployment: An empirical approach’, *Econometrica* **67**(1), 147–162.
- Juster, F. T. and Stafford, F. P. (1991), ‘The allocation of time: Empirical findings, behavioral models, and problems of measurement’, *Journal of Economic Literature* **29**(2), 471–522.
- King, R. G. and Thomas, J. K. (2006), ‘Partial adjustment without apology’, *International Economic Review* **47**(3), 779–809.
- Knabe, A. (2005), Implementing endogenous inside options in Nash wage bargaining models, Unpublished manuscript, Otto-von-Guericke University, Magdeburg.

- Kydland, F. E. and Prescott, E. C. (1982), ‘Time to build and aggregate fluctuations’, *Econometrica* **50**(6), 1345–1370.
- MaCurdy, T. E. (1983), ‘A simple scheme for estimating an intertemporal model of labor supply and consumption in the presence of taxes and uncertainty’, *International Economic Review* **24**(2), 265–289.
- Malcomson, J. (1999), Individual employment contracts, in O. Ashenfelter and D. Card, eds, ‘Handbook of Labor Economics’, Vol. 3B, North Holland, NEW YORK, chapter 35, pp. 2291–2372.
- Merz, M. (1995), ‘Search in the labor market and the real business cycle’, *Journal of Monetary Economics* **36**(2), 269–300.
- Merz, M. and Yashiv, E. (2007), ‘Labor and the market value of the firm’, *American Economic Review* **97**(4), 1419–1431.
- Mortensen, D. and Nagypál, É. (2007), ‘More on unemployment and vacancy fluctuations’, *Review of Economic Dynamics* **10**(3), 327–347.
- Mortensen, D. T. and Pissarides, C. A. (1999), New developments in models of search in the labor market, in O. Ashenfelter and D. Card, eds, ‘Handbook of Labor Economics’, Vol. 3 of *Handbook of Labor Economics*, Elsevier, chapter 39, pp. 2567–2627.
- Nash, J. (1953), ‘Two-person cooperative games’, *Econometrica* **21**, 128–140.
- Nilsen, i. A., Salvanes, K. G. and Schiantarelli, F. (2007), ‘Employment changes, the structure of adjustment costs, and plant size’, *European Economic Review* **51**(3), 577–598.
- Petrongolo, B. and Pissarides, C. A. (2001), ‘Looking into the black box: A survey of the matching function’, *Journal of Economic Literature* **39**(2), 390–431.
- Pissarides, C. A. (2000), *Equilibrium Unemployment Theory*, The MIT Press, Cambridge, Massachusetts.
- Ravn, M. O. and Simonelli, S. (2008), ‘Labor market dynamics and the business cycle: Structural evidence for the United States’, *Scandinavian Journal of Economics* **109**(4), 743–777.
- Rotemberg, J. J. (2006), Cyclical wages in a search and bargaining model with large firms, Working Papers No. 06-5, Federal Reserve Bank of Boston.
- Rubinstein, A. (1982), ‘Perfect equilibrium in a bargaining model’, *Econometrica* **50**(1), 97–109.
- Schelling, T. (1960), *The Strategy of Conflict*, Harvard University Press, Cambridge.
- Shimer, R. (2004), ‘The consequences of rigid wages in search models’, *Journal of the European Economic Association* **2**(2), 469–479.
- Shimer, R. (2005), ‘The cyclical behavior of equilibrium unemployment and vacancies’, *American Economic Review* **95**(1), 25–49.

- Shimer, R. (2007), Reassessing the ins and outs of unemployment, NBER Working Paper Series No. 13421, National Bureau of Economic Research.
- Svejnar, J. (1986), ‘Bargaining power, fear of disagreement, and wage settlements: Theory and evidence from U.S. industry’, *Econometrica* **54**(5), 1055–1078.
- Triest, R. K. (1990), ‘The effect of income taxation on labor supply in the United States’, *Journal of Human Resources* **Vol. 25**(3), 491–516.
- Uhlig, H. (1998), ‘A toolkit for analysing nonlinear dynamic stochastic models easily’, QM&RBC Codes, Quantitative Macroeconomics & Real Business Cycles.
- van Ours, J. and Ridder, G. (1992), ‘Vacancies and the recruitment of new employees’, *Journal of Labor Economics* **10**(2), 138–155.
- Yashiv, E. (2006), ‘Evaluating the performance of the search and matching model’, *European Economic Review* **50**, 909–936.
- Yashiv, E. (2007), ‘Search and matching in macroeconomics’, *European Economic Review* **51**, 1859–1895.

Chapter 2

Endogenous On-the-job Search and Frictional Wage Dispersion

2.1 Introduction

This paper addresses the large degree of frictional wage dispersion found in US data (Hornstein et al., 2007). The standard job matching model without on-the-job search cannot replicate this pattern. With on-the-job search, however, unemployed job searchers are more willing to accept low wage offers since they can continue to seek for better employment opportunities. This explains why observably identical workers may be paid very different wages. Therefore, we examine the quantitative implications of on-the-job search (Nagypál, 2005) in a stochastic job matching model (Mortensen and Pissarides, 1994). Our key result is that the inclusion of *variable* on-the-job search increases the degree of frictional wage dispersion by an order of a magnitude.

Hornstein et al. (2007) study frictional wage dispersion, i.e. wage differentials among observably identical workers, using a “mean-min-ratio” which relates the average wage paid to the lowest wage in the sample. They find that the mean-min-ratio takes values of 1.7 and above in US data. Adopting the framework of Mortensen and Pissarides (1994), the authors then examine the decision problem of an unemployed job searcher assuming that there is no aggregate uncertainty. Each firm-worker pair is characterized by an idiosyncratic productivity level. The worker and the firm form a match if and only if it yields a positive surplus and the wage rate of the worker is determined by Nash bargaining. Hornstein et al. (2007) demonstrate that unemployed job searchers are only willing to accept low wage offers if either (i) the job offer arrival rate is very low or (ii) the expected wage contract is very short. The former case implies that unemployed workers desperately accept any wage offer. Shimer (2007) estimates these values, where, consistent with the model presented, the possibility of direct flows from one employer to another is not taken into account. He finds that 45% of all unemployed workers find a new job within one month, while the average job duration is estimated to be about 2 1/2 years. Using these estimates, Hornstein et al. (2007) find that the model generated

mean-min-ratio is not significantly different from unity.

This paper considers frictional wage dispersion in a model with on-the-job search. This seems to be a natural choice, given that the number of employed workers who change employer each month is about twice as large as the flow of unemployed job searchers into employment (Fallick and Fleischman, 2004). The introduction of direct employment-to-employment¹ transitions has the potential to increase the degree of frictional wage dispersion for two reasons. First, unemployed job searchers who accept (temporarily) a low-wage job offer do not lose the option of labor market search. Hence, they may accept low wage offers for the moment, but continue to seek for better employment opportunities. This implies that a high degree of frictional wage dispersion and a high value of the average job finding rate may coexist at the same time. Second, Nagypál (2008) finds that average wage contract duration is overestimated when employment-to-employment transitions are not taken into account.

In particular, we modify the Mortensen and Pissarides (1994) set up as follows. Each firm-worker pair is characterized by its idiosyncratic productivity level, which is constant throughout the whole duration of the match. Search effort of both employed and unemployed job searchers is endogenous. All firm-worker matches are subject to exogenous and endogenous job separation hazard. Variations in the endogenous job separation margin are driven by aggregate productivity shocks. On-the-job search is motivated by the chance of finding a better job opportunity that promises (i) a higher real wage rate and (ii) a lower hazard of endogenous separation. An unemployed job searcher accepts any job offer, while an employed job searcher accepts a job offer (and quits the old job) only if it includes a higher surplus share.

We calibrate the model in order to match the empirical evidence presented in Nagypál (2008). Accordingly, conditional on not leaving the labor force, about 2/3 of all workers who separate from their employer are matched with a new one in the following month. In addition to that, the assumption of variable on-the-job search *and* endogenous job separation shocks helps us to replicate the observation that on-the-job search is most intense among workers in low-wage matches close to the separation margin (Fallick and Fleischman, 2004; Christensen et al., 2005). Thus, our model is consistent with the empirical observation that (i) employment-to-employment transition rates are highest in matches that pay low wages and (ii) the aggregate employment-to-employment transition rate is slightly below the value of 3%.

High employment-to-employment transition rates imply that low-wage matches exhibit a high option value of labor market search. This stimulates unemployed job searchers to accept such offers and to search on-the-job for better employment opportunities. Moreover, we note that the expected duration of low-wage job matches is far below the aver-

¹Our paper addresses only transitions of workers from one employer to another. We do not consider promotions within a given firm.

age. Consequently, our calibrated model is able to generate a mean-min-ratio equal to 1.3. Compared to the value provided by Hornstein et al. (2007), the percentage difference between the average and the lowest wage paid rises by an order of a magnitude. Therefore, we conclude that the introduction of variable on-the-job search into a model with endogenous job separations is an effective means to generate frictional wage dispersion.

In addition, we examine the dynamic behavior of the model at the business cycle frequencies. Our analysis focuses on the cyclical movements of vacancies, unemployment, and the real wage rate. Interest in this issue has been sparked by the influential paper of Shimer (2005), which states that the job matching model with exogenous separations (Pissarides, 1985) is not able to replicate the high degree of labor market volatility. Furthermore, Mortensen and Nagypál (2008) point out that counter-cyclical fluctuations in the endogenous separation margin are able to amplify the variations in the number of unemployed job searchers on the one hand. On the other hand, these strong counter-cyclical movements provide firms incentives to open more vacancies during economic downturns. Hence, the model generated Beveridge curve may be counter-factually positively sloped. In our model, on the contrary, variable on-the-job search uncouples aggregate search effort from the number of unemployed job searchers. Since on-the-job search is pro-cyclical, we note that total search effort, i.e. the sum of all effort undertaken by employed and unemployed job searchers, is relatively stable over the business cycle. This stimulates firms to open more vacancies when aggregate productivity is high. As a consequence, our model is able to replicate a negatively sloped Beveridge curve.

The remainder of this paper is organized as follows. Section 2.2 presents the model environment. Section 2.3 calibrates the model and evaluates its quantitative performance against US data. Section 2.4 concludes.

2.2 The Model Environment

2.2.1 Employment Relationships

There is a continuum of ex-ante identical workers in the economy, having unit mass and a continuum of potential firms, having infinite mass. Both firms and workers are risk-neutral. Production takes place in one-firm-one-worker matches. Each active match produces output according to a linear technology: $y(a, t) = az_t$. We assume that match-specific idiosyncratic productivity level a is constant throughout the whole duration of the match. The exogenous distribution of a is described by the cumulative distribution function $P(a)$ with support $[0, \infty)$. Aggregate productivity z_t , instead, is subject to an exogenous shock specified by following autoregressive process:

$$\ln(z_t) = (1 - \varrho) \ln(\bar{z}) + \varrho \ln(z_{t-1}) + \epsilon_t \text{ with } \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2) \text{ and } iid. \quad (2.1)$$

2.2.2 The Labor Market

In the beginning of period t , there are N_t matched firm-worker pairs. The endogenous distribution of firm-worker pairs over idiosyncratic productivity levels is described by a cumulative distribution function $G(a, t)$ with support $[0, \infty)$. All firm-worker pairs face exogenous separation with probability ρ^x . In addition, a match may be separated if its idiosyncratic productivity level is below the current reservation productivity $a_{r,t}$. Workers who lose their job, whether for exogenous or endogenous reasons, immediately enter the period t matching pool. Hence, the unemployment rate U_t is given by the share of workers who are not engaged in *active* employment relationships:²

$$U_t = 1 - N_t(1 - \rho^x) \int_{a_{r,t}}^{\infty} g(a, t) da.$$

Labor market search takes place parallel to production. The pool of job searchers comprises all unemployed and all employed workers who search on-the-job for better employment opportunities. Search effort of both, unemployed and employed job searchers, is endogenous. Unemployed job searchers are identical and, hence, search all with the same intensity $e_{u,t}$ on the labor market. The search effort of an employed worker $e_w(a, t)$ depends on her idiosyncratic productivity level a . Thus, aggregate search effort E_t is given by:

$$E_t = U_t e_{u,t} + N_t(1 - \rho^x) \int_{a_{r,t}}^{\infty} e_w(a, t) g(a, t) da.$$

Search effort of unemployed and employed job searchers incurs a cost. In particular, we assume that the respective cost functions are given as:

$$c[e_{u,t}] = \zeta_u [e_{u,t}]^{\phi_u}, \quad c[e_w(a, t)] = \zeta_w [e_w(a, t)]^{\phi_w},$$

where the parameter ϕ_i , $i = u, w$ captures the fact that the level and the curvature of the search cost function may depend on the employment status.

Besides the pool of job searchers, the period t matching market consists of the aggregate number of vacancies V_t . Firms with unfilled positions may decide whether or not to post a vacancy, where posting a vacancy entails a cost κ per period. Free entry into the matching market determines the aggregate number of posted vacancies.

New matches are formed at the end of period t . The number of newly formed firm-worker pairs is given by a Cobb-Douglas matching function with constant returns to scale. This function relates aggregate job matches M_t to aggregate vacancies V_t and aggregate search effort E_t :

$$M(V_t, E_t) = \min \left[\chi V_t^\gamma E_t^{1-\gamma}, V_t, 1 \right],$$

where χ is a constant scaling factor.

²Our model abstracts from movements into and out of the labor force.

The ratio between aggregate vacancies and aggregate search effort measures the tightness of the labor market. By linear homogeneity of the matching function, the matching probability per unit search effort $f(\theta_t)$, and the matching rate per vacancy $q(\theta_t)$, respectively, depend solely on the value of market tightness θ_t :

$$f(\theta_t) = \frac{M(V_t, E_t)}{E_t} = \chi \left(\frac{V_t}{E_t} \right)^\gamma = \chi \theta_t^\gamma, \quad (2.2)$$

$$q(\theta_t) = \frac{M(V_t, E_t)}{V_t} = \chi \left(\frac{V_t}{E_t} \right)^{\gamma-1} = \chi \theta_t^{\gamma-1}. \quad (2.3)$$

The tighter the labor market, the longer the expected time to fill a vacancy, but the shorter the expected search for a job (and vice versa). The fact that firms and households do not internalize these adverse effects on the aggregate return rates gives rise to congestion externalities.

Labor market search and match formation entails that the employment distribution in the beginning of period $t + 1$ is given by:

$$\begin{aligned} N_{t+1} \int_0^a g(a, t+1) da &= \int_0^a \left\{ U_t f(\theta_t) e_{u,t} p(a) + \right. \\ &\quad I_{a > a_{r,t}} (1 - \rho^x) (1 - \tau(a, t)) N_t g(a, t) + \\ &\quad \left. (1 - \rho^x) N_t f(\theta_t) \left(\int_{a_{r,t}}^a e_w(a', t) g(a', t) da' \right) p(a) \right\} da, \end{aligned} \quad (2.4)$$

where I is an indicator function equal to zero if a is below the current reservation productivity $a_{r,t}$, and otherwise equal to one. The right hand side of equation (2.4) is made up of (i) the mass of unemployed job searchers who succeed in meeting an employer, (ii) the distribution of workers in existing job matches that experience neither job separation nor a transition (with probability $1 - \tau(a, t)$) to a new employer and, (iii) the distribution of new job matches established by successful employment-to-employment transitions. Evaluating equation (2.4) at $a \rightarrow \infty$ yields a more familiar law of motion:

$$N_{t+1} = (1 - \rho^x) N_t [1 - G(a_{r,t})] + f(\theta_t) e_{u,t} U_t.$$

2.2.3 The Joint Surplus

Let the value of unemployment to a worker be \mathcal{U}_t , the value of a vacancy to a firm be \mathcal{V}_t , the value of a match to a worker be $\mathcal{W}(a, t)$, and the value of a match to a firm be $\mathcal{J}(a, t)$. Hence, the joint surplus of an active match $\mathcal{S}(a, t)$ is given by the value of output $y(a, t)$, net of expenses for search on-the-job $c[e_w(a, t)]$ and the joint outside alternative $(\mathcal{U}_t + \mathcal{V}_t)$, plus the joint continuation value of the current match $\mathcal{C}(a, t)$ in the future:

$$\mathcal{S}(a, t) = y(a, t) - c[e_w(a, t)] - (\mathcal{U}_t + \mathcal{V}_t) + \mathcal{C}(a, t). \quad (2.5)$$

The joint surplus is divided by Nash (1953) bargaining. Consequently, the firm gains period-by-period the fixed portion ξ , while the share $1 - \xi$ is allocated to the worker. We assume that wage bargaining takes place in the beginning of period t , but *after* the employed worker has made her search decision. This timing assumption has two important implications. First, employed workers who have just made a transition to a new employer are not able to resume their old position. Therefore, the outside alternative of all workers is equal to the value of unemployment. Second, the bargaining outcome does not influence the search decision of the worker. Thus, the surplus shares are given as:³

$$\xi \mathcal{S}(a, t) = \mathcal{J}(a, t) - \mathcal{V}_t, \quad (1 - \xi) \mathcal{S}(a, t) = \mathcal{W}(a, t) - \mathcal{U}_t. \quad (2.6)$$

Equation (2.6) shows that the surplus share of the firm equals the value of an active match to the firm $\mathcal{J}(a, t)$ net of the value of an unfilled vacancy \mathcal{V}_t . The surplus share of the worker equals the value of an active match to the worker $\mathcal{W}(a, t)$ net of the value of unemployment \mathcal{U}_t . Furthermore, the Nash sharing rule implies that any endogenous separation decision is made by mutual consent. Consequently, the current reservation productivity $a_{r,t}$ has to satisfy the following job separation condition:

$$\mathcal{S}(a_{r,t}, t) = 0 \Leftrightarrow y(a_{r,t}, t) - c[e_w(a_{r,t}, t)] + \mathcal{C}(a_{r,t}, t) = \mathcal{U}_t + \mathcal{V}_t.$$

2.2.4 The Problem of an Unemployed Job Searcher

We now examine the decision problem of an unemployed job searcher. There is a continuum of ex-ante unemployed job searchers in the economy, having mass U_t . All unemployed job searchers take the aggregate matching rate per unit search effort f_t as given and search $e_{u,t}$ units on the labor market. Hence, an unemployed job searcher can expect to meet a firm at the rate:

$$\tilde{f}(e_{u,t}, \theta_t) = f_t e_{u,t}. \quad (2.7)$$

When an unemployed job searcher meets a firm at the end of period t , the pair draws its idiosyncratic productivity level a , which is constant throughout the whole duration of the match. Given that next period's reservation productivity $a_{r,t+1}$ is still unknown, unemployed job searchers accept every match for the moment. If the match survives

³The present timing assumption is clearly a simplification. However, to our knowledge, there is no study investigating all aspects of a multi-player bargaining game under variable on-the-job search. Cahuc et al. (2006) examine the implications of a multi-player bargaining game, where the search intensity of employed job searchers is constant (Burdett and Mortensen, 1998). They demonstrate that between-firm competition increases the average wage rate if employed job searchers can resume their old positions. Papp (2009) examines the implications of this approach in a general equilibrium model. On the other hand, Shimer (2006) analyzes a strategic bargaining game between an employed job searcher and a single firm, where the firm is willing to offer a higher wage rate in order to reduce the extent of transitions to other employers. Indeed, the examination of these questions in a stochastic environment is beyond the scope of this paper.

exogenous destruction at the beginning of period $t+1$, the firm and the worker observe the realization of aggregate productivity z_{t+1} and decide upon endogenous separation. Only if the idiosyncratic productivity of the firm-worker match is greater than the reservation productivity $a > a_{r,t+1}$, the match will become active. Thereby, the reservation strategy of the worker implies that she gains at least the value of unemployment \mathcal{U}_{t+1} . This setup implies that the expected value of a match to the worker is given by:

$$\bar{\mathcal{W}}_t = \int_0^\infty \mathcal{W}_t(a)p(a)da, \quad (2.8)$$

where $\mathcal{W}(a, t) = \mathcal{U}_t$ below the reservation productivity $a_{r,t}$. Hence, $\mathcal{W}(a, t)$ captures not only the value of active matches, but also the value of matches that are endogenously terminated. For this reason, the value of unemployment can be written as:

$$\mathcal{U}_t = \max_{e_{u,t} \geq 0} \left\{ b - c[e_{u,t}] + \beta E_t \left[(1 - \rho^x) \tilde{f}_t \left(\bar{\mathcal{W}}_{t+1} - \mathcal{U}_{t+1} \right) + \mathcal{U}_{t+1} \right] \right\}. \quad (2.9)$$

Each worker who is not engaged in an active employment relationship receives the flow income b , net of $c[e_{u,t}]$ units search costs. Future utility is discounted at the rate β . The worker expects to find an average job in period $t+1$ with probability $(1 - \rho^x) \tilde{f}_t$. In this case, the worker gains the expected surplus share $(\bar{\mathcal{W}}_{t+1} - \mathcal{U}_{t+1})$. In addition, the unemployed worker will receive at least the value of unemployment \mathcal{U}_{t+1} – independent of whether or not she succeeds in finding an active employment relationship.

Consequently, the search effort choice $e_{u,t}$ of an unemployed worker has to satisfy following first order condition:

$$c' [e_{u,t}] = (1 - \rho^x) f_t \beta E_t \left\{ \bar{\mathcal{W}}_{t+1} - \mathcal{U}_{t+1} \right\}. \quad (2.10)$$

The first order condition of an unemployed worker states that the marginal costs of labor market search (the left hand side) must be equal to the marginal benefits (the right hand side). The latter is given by the matching rate per unit labor market search f_t , the exogenous destruction rate ρ_x , and the expected present value of the worker's surplus share.

2.2.5 The Problem of an Employed Job Searcher

Employed workers who search on-the-job for better employment opportunities face the same meeting rate per unit search effort f_t as unemployed job searchers. The search effort of an employed worker, however, depends on her idiosyncratic productivity level a . Thus, the expected meeting rate of an employed job searcher is given as: $f_t e_w(a, t)$. Moreover, the reservation strategy of an employed worker implies that she accepts matches at the end of period t only if the idiosyncratic productivity of the new match \hat{a} is strictly larger

than the one of the old match a . Otherwise, she remains on her old job. Given that the idiosyncratic productivity level of new matches is drawn from the exogenous distribution $P(a)$, the current setting implies that an employed job searcher of type a rejects new job offers at the rate $P(a)$. Hence, the expected employment-to-employment transition rate of an employed worker with idiosyncratic productivity a reads as:

$$\tau(a, t) = f_t e_w(a, t) [1 - P(a)]. \quad (2.11)$$

Consequently, the worker's value of a match with idiosyncratic productivity a can be written as:

$$\begin{aligned} \mathcal{W}(a, t) = \max_{e_w(a, t) \geq 0} & \left\{ w(a, t) - c[e_w(a, t)] + \beta E_t \left\{ \right. \right. \\ & \left(\left\langle (1 - \tau(a, t))(1 - \rho^x) (\mathcal{W}(a, t+1) - \mathcal{U}_{t+1}) \right\rangle + \right. \\ & \left. \left. \left\langle f_t e_w(a, t)(1 - \rho^x) \int_a^\infty (\mathcal{W}(\hat{a}, t+1) - \mathcal{U}_{t+1}) p(\hat{a}) d\hat{a} \right\rangle + \mathcal{U}_{t+1} \right) \right\} \right\}. \end{aligned} \quad (2.12)$$

The worker's value of a match $\mathcal{W}(a, t)$ is given by the real wage rate $w_{a,t}$, net of expenses for search on-the-job $c[e_w(a, t)]$, and the continuation value of the match to the worker. The worker's continuation value is given by (i) the expected surplus share of the *current* employment relationship, (ii) the expected surplus share of a prospective job opportunity with higher ($\hat{a} > a$) idiosyncratic productivity, and (iii) the expected present value of unemployment (the minimum gain of the worker). The first term (i) refers to the worker who does not find a better employment opportunity, but suffers neither exogenous nor endogenous job destruction. The second term (ii) characterizes the worker who succeeds in finding a better job opportunity which survives exogenous and endogenous job destruction. The third term (iii) applies if the *current* or the *prospective* employment relationship is terminated either for *exogenous* or *endogenous* reasons.

When employed workers make their search effort decision, they anticipate the Nash wage rate $w(a, t)$ correctly. Wage bargaining, however, does not start until the search effort decision has been made.⁴ Hence, the optimal search intensity of an employed worker with idiosyncratic productivity a is given by following first order condition:

$$c'[e_w(a, t)] = f_t (1 - \rho^x) \beta E_t \left\{ \int_a^\infty (\mathcal{W}(\hat{a}, t+1) - \mathcal{W}(a, t+1)) p(\hat{a}) d\hat{a} \right\}. \quad (2.13)$$

Equation (2.13) shows that the search intensity of employed workers decreases in the idiosyncratic productivity level a . The expected upgrading value $\mathcal{W}(\hat{a}, t+1) - \mathcal{W}(a, t+1)$, which is determined by the expected surplus from a prospective job opportunity \hat{a} given

⁴See Footnote (3).

the value of the current job a , is highest if the idiosyncratic productivity level a is slightly above the current reservation productivity level $a_{r,t}$. Workers engaged in these matches fear to loose their job in the case of a negative shock to aggregate productivity.

2.2.6 Optimal Vacancy Posting

There is an infinite mass of firms with unfilled positions. Each firm with an unfilled position may decide whether or not to post a vacancy. Equation (2.3) shows that a firm with an open vacancy expects to meet a job searcher at rate $q(\theta_t)$. However, the firm anticipates the possibility of meeting an employed job searcher who might reject the job offer in favor of her old job. Hence, the probability of *filling* a vacancy \tilde{q}_t is given by the probability of *meeting* a worker $q(\theta_t)$, the job offer acceptance rate $(1 - \rho_t^j)$, and the exogenous separation rate ρ^x :

$$\tilde{q}_t = q(\theta_t) (1 - \rho_t^j) (1 - \rho^x), \quad (2.14)$$

where the share of rejected job offers ρ_t^j is given by the number of rejected job offers divided by the number of aggregate matches:

$$\rho_t^j = \left(N_t (1 - \rho^x) f_t \int_{a_{r,t}}^{\infty} e_w(a, t) P(a) g(a, t) da \right) / M_t. \quad (2.15)$$

Since next period's reservation productivity is still unknown, the firm accepts matches of any idiosyncratic productivity level a for the moment. When the worker and the firm decide whether to engage in production in period $t + 1$, the reservation strategy of the firm ensures that the value of the match is greater or equal to the value of an unfilled vacancy \mathcal{V}_t . Hence, the expected value of a filled vacancy is given by:

$$\overline{\mathcal{J}}_t = \int_0^{\infty} \mathcal{J}(a, t) p(a) da, \quad (2.16)$$

where the value of a match to the firm $\mathcal{J}(a, t)$ can be written as:

$$\mathcal{J}(a, t) = y(a, t) - w(a, t) + \beta E_t \{ (1 - \tau(a, t)) (1 - \rho^x) \mathcal{J}(a, t + 1) \}. \quad (2.17)$$

The firm enjoys the value of production, net of labor costs $w(a, t)$. The firm's continuation value is determined by the current employment-to-employment transition rate $\tau(a, t)$, the exogenous job destruction rate ρ^x , and the expected present value of the current match in the next period. Furthermore, we note that $\mathcal{J}(a, t) = \mathcal{V}_t$ below the reservation productivity $a_{r,t}$. Hence, $\mathcal{J}(a, t)$ measures not only the value of active matches, but also the value of matches that are consensually terminated. Hence, the value of an unfilled

vacancy is given by:

$$\mathcal{V}_t = -\kappa + \beta E_t [\tilde{q}_t \bar{\mathcal{J}}_{t+1} + (1 - \tilde{q}_t) \mathcal{V}_{t+1}]. \quad (2.18)$$

Recall that posting a vacancy entails a cost κ per period. Therefore, the firm expects to gain the value of a filled vacancy $\bar{\mathcal{J}}_t$ with probability \tilde{q}_t . Otherwise, the vacancy remains unfilled. Free entry into the matching market ensures that the firm's outside option, i.e. the value of an unfilled vacancy, is zero in every period: $\mathcal{V}_t = 0$. Thus, the number of posted vacancies has to satisfy following condition:

$$\frac{\kappa}{\tilde{q}_t} = \beta E_t \bar{\mathcal{J}}_{t+1}. \quad (2.19)$$

2.2.7 The Wage Function

Using the Nash sharing rule (2.6), the value of a match to the worker (2.12), and the value of a match to the firm (2.17), the joint surplus of the match (2.5) can be rewritten as follows:

$$\begin{aligned} \mathcal{S}(a, t) = & y(a, t) - c[e_w^*(a, t)] - \mathcal{U}_t - \mathcal{V}_t + \\ & \beta E_t \left\{ \left\langle (1 - \tau(a, t))(1 - \rho^x) \mathcal{S}(a, t + 1) \right\rangle + \right. \\ & \left. \left\langle f_t e_w(a, t)(1 - \rho^x) \int_a^\infty (1 - \xi) \mathcal{S}(\hat{a}, t + 1) p(\hat{a}) d\hat{a} \right\rangle + \mathcal{U}_{t+1} \right\}. \end{aligned} \quad (2.20)$$

where $e_w^*(a, t)$ satisfies equation (2.13).

Equation (2.20) shows that the joint continuation value of the match consists of (i) the option value of the current match, (ii) the option value of on-the-job search, and (iii) the option value of unemployment. Thereby, only the benefits of on-the-job search going to the worker $(1 - \xi) \mathcal{S}(\hat{a}, t + 1)$ enter the mutual surplus, while the benefits going to the new employer $\xi \mathcal{S}(\hat{a}, t + 1)$ are not taken into account. Furthermore, the Nash bargaining solution entails following real wage function, depending on the idiosyncratic productivity level a :

$$\begin{aligned} w(a, t) = & (1 - \xi)y(a, t) + \xi (c[e_w^*(a, t)] + \mathcal{U}_t) - \\ & \xi \beta E_t \left\{ \left\langle f_t e_w(a, t)(1 - \rho^x) \int_a^\infty (1 - \xi) \mathcal{S}(\hat{a}, t + 1) p(\hat{a}) d\hat{a} \right\rangle + \mathcal{U}_{t+1} \right\}. \end{aligned} \quad (2.21)$$

The real wage rate is given by the weighted average of (i) the value of production and (ii) the value of unemployment, plus compensation for search on-the-job, minus the option value of on-the-job search to the worker. The positive option value to the worker reduces her reservation wage. Therefore, we observe that the Nash wage in our model

with on-the-job search is lower than in the baseline matching model.

2.2.8 Equilibrium

The competitive equilibrium is given by the unemployment rate U_t , the aggregate number of vacancies V_t , the search effort of unemployed workers $e_{u,t}$, the function of employed workers' search effort $e_w(a, t)$, the distribution of firm-worker matches $G(a, t)$, and a wage function $w(a, t)$, such that:

- \mathcal{U}_t and $\mathcal{W}(a, t)$ are the value of unemployment and of a match, respectively, for workers making optimal search effort decisions, given U_t , V_t , E_t , $w(a, t)$, and $G(a, t)$. $e_w(a, t)$ and $e_{u,t}$ are the corresponding optimal search effort policies.
- \mathcal{V}_t and $\mathcal{J}(a, t)$ are the value of a vacancy and of a match for firms making optimal vacancy creation decisions, given U_t , V_t , E_t , $w(a, t)$, $e_w(a, t)$, $e_{u,t}$, and $G(a, t)$.
- Total factor productivity z_t follows the exogenous stochastic process (2.1).
- There is free entry into the matching pool of vacancies.
- New firm-worker matches draw their idiosyncratic productivity a from an exogenously given distribution $P(a)$.
- Wages are set by sharing the surplus of an active firm-worker match in fractions ξ and $1 - \xi$, respectively, given the wage function $w(a, t)$.
- The distribution $G(a, t)$, the unemployment rate U_t , the aggregate number of vacancies V_t , and the total search effort E_t are consistent with the decisions of the agents in the economy.
- An initial condition for the share of matched firm worker pairs N_0 is given.

2.3 Model Evaluation

2.3.1 Computational Issues

We analyze the cyclical properties of the model economy by value function iteration on a discrete state space. Thereby, the treatment of the endogenous productivity distribution as an endogenous state variable establishes a computational challenge. Particularly, as in the case of endogenous separations the endogenous productivity distribution exhibits a discontinuity at the reservation productivity $a_{r,t}$. This might be the reason why only a small number of authors so far have addressed the issue of on-the-job search in a job matching model. Among others, Nagypál (2005) examines the stationary equilibrium of

a job matching model with variable on-the-job search, where idiosyncratic productivity shocks lead to endogenous separations. She concludes that workers close to the endogenous separation margin show the highest intensity of on-the-job search. In a companion paper, Nagypál (2007) considers a log-linear approximation around the non-stochastic steady state. However, in order to ensure differentiability of the endogenous distribution, the model drops the assumption of endogenous separations. On the contrary, Fahr (2007) argues that firm-worker pairs are subject to noisy signals about the idiosyncratic productivity level. This implies that some profitable matches separate endogenously by mistake, while some non-profitable matches are continued. Thus, the discontinuity at the reservation productivity may be smoothed out by a logistic distribution. For this reason, the model with on-the-job search and endogenous separations can be solved by a linear approximation. Tasci (2007) extends the model by Pries and Rogerson (2005) in order to allow for *exogenous* on-the-job search. In this setting, it suffices to approximate the worker's acceptance probability. For this purpose, the algorithm by Krusell and Smith (1998) is utilized. Krause and Lubik (2007) assume segregated markets for good and bad jobs, where separations are exogenous and constant in both market segments. Finally, Menzio and Shi (2009) introduce on-the-job search into a model of directed search akin to Moen (1997). They demonstrate that, in this setting, the agents' decisions are independent of the endogenous productivity distribution.

Instead, our approach is based on the observation that, in the US, the half life deviation of the actual unemployment rate from its stationary value is close to one month (Elsby et al., 2009). Consequently, the correlation between the stationary rate and the actual unemployment rate in the following month is very close to unity (Shimer, 2007). Since the lag is so short, Hall (2009) suggests neglecting the fact that the unemployment rate is governed by a backward-looking law of motion. In addition, as described above, we note that the flow of workers from one employer to another is more than twice as large as the flow from unemployment to employment. For this reason, we treat neither the unemployment rate nor the corresponding endogenous productivity distribution as endogenous state variables. The only state variable in our model is aggregate productivity.

2.3.2 Calibration

In order to capture the high transition rates in the US economy, we calibrate the model so that one period corresponds to one month. When simulating the model, we time-aggregate the artificial series to quarterly data and evaluate it against the quarterly US time series. Table 2.1 summarizes the parameter values of our model.

Preferences Workers are risk-neutral and supply labor inelastically. The discount factor β is chosen to match an annual real interest rate of 4 percent (Kydland and Prescott,

1982). Furthermore, estimates by Christensen et al. (2005) and Yashiv (2000) for employed and unemployed job searchers, respectively, indicate that both types of job searchers face a quadratic cost of search effort, i.e. $\phi_e = \phi_u = 2$ and $\zeta_e = \zeta_u = 1/2$.

Matching and the Labor Market We target an average unemployment rate $U = 10\%$ and a workers' meeting rate $\tilde{f}_t = 27\%$ (Hall, 2005). These figures refer to the officially unemployed job searchers plus the pool of marginally attached non-participants (Jones and Riddell, 1999). Our calibrated value of unemployment benefits $b = 0.85$, together with the implied average value of search disutility ($c[\bar{e}_w^*] = 0.14$) and average output per worker $\bar{y} = 1.25$, yields a replacement rate equal to 0.56. This value lies within the range found in the literature.⁵ Targets for the transition rate out of employment are provided by Hall (2005). Therefore, we set the exogenous separation rate ρ^x to 0.0275. Furthermore, our calibration implies that the endogenous separation rate ρ^n is close to 0.01, and the average employment-to-employment transition rate $\bar{\tau}$ is equal to 0.04. Beside that, we choose the firm's bargaining power $\xi = 0.25$ and the per-period vacancy posting cost $\kappa = 0.06$, such that the unemployment rate and the percentage of vacancy posting costs in aggregate output $(\kappa V)/Y = 1.9\%$ are in line with our targets. The latter value implies that the steady-state asset value of a match to the firm is equal to 19% of average output (Yashiv, 2000). Finally, we calibrate the two parameters of the matching function. First, we set the matching function constant χ equal to 0.45.⁶ Second, we calibrate the matching elasticity of vacancies η equal to 0.5, which is within the plausible range $[0.3 - 0.5]$ proposed by Petrongolo and Pissarides (2001).

Productivity We approximate the stochastic process (Equation 2.1) with a discrete valued first-order Markov process using the method by Tauchen (1986). Thereby, we use the values ($\varrho = 0.97, \sigma_\epsilon = 0.007$) suggested by Hagedorn and Manovskii (2008).⁷ The number of grid points in the state space is set equal to 29. The exogenous productivity distribution is assumed to be log-normal with mean $\mu = 1$ and a standard deviation equal to $\sigma_p = 0.1$. We compute the exogenous and the endogenous productivity distribution, $P(a)$ and $G(a)$ respectively, on a very fine grid with 500 points between 0.7 and 1.5.

⁵The value used by Shimer (2005), $b = 0.4$, comprises only pecuniary benefits and, thus, is considered to be a lower bound. Values beyond are usually justified by the reference to "leisure gain from unemployment". Angerhausen et al. (2009) provide a microfoundation for this claim, while the results of Costain and Reiter (2008) indicate that this "gain" is quantitatively important. An upper bound is provided by the estimate $b = 0.95$ of Hagedorn and Manovskii (2008), who attempt to match the elasticity of the real wage rate.

⁶As argued by Shimer (2005), the model allows the normalization of this value.

⁷The published version of the model is calibrated so that one period corresponds to one week. Please refer to the working paper version (available at: <http://www.econ.umn.edu/macro/2005/hagedorn.pdf>) for the monthly calibration.

2.3.3 Steady State Analysis

The Endogenous Productivity Distribution Figure (2.1a) displays the probability density function of the exogenous productivity distribution $P(a)$, represented by a blue line, as well as the *endogenous* productivity distribution at 5 different levels of aggregate productivity (at grid points 1, 8, 15, 22, and 29), *after* endogenous separation has taken place. Grid point 15 (turquoise line) represents the stochastic steady state. The graph clearly shows that the positive selection of employed workers towards matches with higher idiosyncratic productivity shifts the endogenous distribution to the right. In the steady state, average match quality is about 25% higher than the mean of the exogenous productivity distribution. Moreover, we note that the impact of aggregate productivity on average match quality follows a non-linear pattern (Figure 2.1b). The slope of the right tail is much steeper than the slope of the left tail. This suggests that the channel from aggregate productivity to average match quality is shaped by two opposing effects. On the one hand, recessions destroy low-quality matches which are likely to be profitable during economic upswings. This is referred to as the “cleansing” effect of recessions (Caballero and Hammour, 1994). On the other hand, we observe that search intensity of on-the-job searchers is pro-cyclical (Figure 2.1d). Consequently, the number of employment-to-employment transitions is pro-cyclical as well (Figure 2.1f). This implies that recessions are times when workers tend to stay in low quality matches. Barlevy (2002) refers to this as the “sullyng” effect of recessions. In our calibrated model, the “sullyng” effect is dominant. Therefore, average match quality is pro-cyclical. Nevertheless, the flat shape of the left tail shows that also the “cleansing” effect is present.

The Average Real Wage Rate The evolution of average match quality over the business cycle has important implications for the dynamic behavior of the real wage rate. In the presence of period-by-period Nash bargaining, both variables are linked very closely. This is manifested by Figure 2.1b. Individual wages, instead, increase monotonously in aggregate productivity (Figure 2.1g). This finding indicates that our model may be able to match an important empirical observation. Several authors, among others Solon et al. (1994), Bowlus et al. (2002) and Hart (2006), report that individual wage profiles are strongly pro-cyclical. Aggregate data, however, show (at most) a weakly pro-cyclical pattern. The literature argues that the apparent discrepancy is due to the fact that aggregate data often neglects the cyclical variations in the average match quality. Our model seems to be consistent with this claim.

Frictional Wage Dispersion Furthermore, we observe that the real wage rate paid in the least productive active match is essentially independent of aggregate productivity (Figure 2.1g). Consequently, the ratio between the average and the lowest wage paid in the economy, the so-called “mean-min-ratio”, follows a similar pattern as the average

real wage rate (Figure 2.1h). The mean-min-ratio measures the degree of frictional wage dispersion, i.e. wage differentials among ex-ante identical workers. Hornstein et al. (2007) demonstrate that the job matching model without on-the-job search is unable to match the high degree of frictional wage dispersion found in the data. In the US, the average wage paid to similar workers is about 70% higher than the lowest wage within the set. In contrast, the mean-min ratio generated by a reasonably calibrated version of the job matching model without on-the-job search is not significantly different from unity. In our calibrated model, the steady state value of the mean-min-ratio is equal to 1.4. Thus, our model is able to explain about one-half of the empirical degree of frictional wage dispersion.

Hornstein et al. (2007) argue that the inclusion of on-the-job search is essential for generating these results. In the presence of high job finding rates, unemployed job searchers have no incentive to accept a long-term job that pays a low wage rate. With on-the-job search, employed workers are more likely to accept low wage offers for the moment and continue to seek for better employment opportunities. However, a model with exogenous on-the-job search (Burdett and Mortensen, 1998) requires far too high EE -transition rates in order to replicate the degree of frictional wage dispersion found in the data. For this reason, we introduce variable on-the-job search and endogenous separations. These two modifications allow us to concentrate on-the-job search among workers in low quality matches. These workers have (i) a large probability of finding a better employment opportunity and (ii) a high probability of entering unemployment. Therefore, our model is able to generate significant frictional wage dispersion. At the same time, the share of employed workers who change employer each month is very close to the empirical value of 3% (Figure 2.1f).

On-the-Job Search Figure (2.1c) illustrates the search effort decision of employed and unemployed job searchers, respectively. The horizontal lines represent the effort per unemployed job searcher, which depends only on the level of aggregate productivity. The search effort choice of employed job searchers, instead, is a function of aggregate and idiosyncratic productivity. As expected, search effort of any type of worker rises with aggregate productivity, while search effort of employed job searchers falls with idiosyncratic productivity. This happens for two reasons. First, the probability of finding a better employment opportunity is smaller the higher the quality of the current match. Second, the hazard of endogenous job separation decreases with the distance to the current reservation productivity. We observe that the search behavior of an employed worker at the reservation productivity (the “marginal” worker) is almost identical to the search behavior of an unemployed. Due to the low reservation productivity, these workers are (i) likely to find a better employment opportunity and (ii) face a huge endogenous separation hazard in the case of a negative productivity shock. Moreover, we note that the bulk of employed

workers, i.e. workers in matches with idiosyncratic productivity $a > 1.1$, do not make any significant effort to find a new employer. This observation is consistent with the findings by Fallick and Fleischman (2004) and Christensen et al. (2005).

Total search effort, i.e. the sum of all effort undertaken by employed and unemployed job searchers, seems to be relatively stable (Figure 2.1d). The composition, however, changes considerably over the business cycle. Search effort of employed job searchers rises with aggregate productivity, since there are more on-the-job searchers (extensive margin) who search more than the employed workers already in place (intensive margin). In particular, note that the search effort of a given employed worker increases only by little when economic conditions improve. This indicates that escaping from imminent unemployment is probably the main motive for on-the-job search. In addition to that, we observe that aggregate search effort of unemployed job searchers is counter-cyclical. Even though the search intensity per unemployed job searcher rises (intensive margin, see Figure 2.1c), the number of unemployed job searchers declines sharply during economic upswings (extensive margin, see Figure 2.2b). This implies that a firm is much more likely to meet an employed job searcher when aggregate productivity is high.

Employment-to-Employment Transitions Figure (2.1e) presents the rates at which employed workers find new employers. Given that the meeting rate per unit search effort is not very elastic, EE -transition rates follow the same pattern as search intensity of employed workers. Consequently, marginal workers enjoy almost the same transition probability (well above 30%) as unemployed job searchers during economic booms (grid point 29). During recessions (grid point 1), on the contrary, the EE -transition rate of marginal workers is only slightly above 10%. The job finding rate of unemployed job searchers, on the other hand, never falls below a value of 25%. The huge differences in the transition probabilities of marginal workers may help to explain the shape of the endogenous productivity distribution at the reservation productivity (Figure 2.1a). When aggregate productivity is low, the endogenous productivity distribution exhibits a clear cut-off point. Yet, when aggregate productivity is high, we observe that the endogenous productivity distribution is very smooth.

Aggregate Labor Market Dynamics In addition, our model allows analyzing the cyclical behavior of the labor market. Interest in this issue has been sparked by the influential paper of Shimer (2005). The survey paper by Yashiv (2008) provides a general picture of US labor market dynamics over the business cycle. Accordingly, gross worker flows between employment and unemployment are counter-cyclical and volatile. On the one hand, counter-cyclical flows from employment to unemployment are driven by counter-cyclical movements in the job separation margin (Fujita, 2009). On the other hand, counter-cyclical flows from unemployment to employment are due to the fact that

the percentage fall in the job finding rate is smaller than the percentage rise in the unemployment rate in the aftermath of a negative productivity shock (Fallick and Fleischman, 2004). This implies that the number of unemployed job searchers is strongly counter-cyclical. As pointed out by Mortensen and Nagypál (2008), such a scenario might give firms incentives to open more vacancies during economic downturns. Hence, the model generated Beveridge curve may be counter-factually positively sloped. In the case of variable on-the-job search, however, total search effort is uncoupled from the number of unemployed job searchers. For this reason, strong counter-cyclical movements in the aggregate unemployment rate do not necessarily induce counter-cyclical variations in the number of posted vacancies.

Figure (2.2a) shows that gross worker flows between employment and unemployment are, indeed, counter-cyclical.⁸ The flow of employed workers into unemployment is determined by exogenous and endogenous separation (Figure 2.2d).⁹ The graph depicting the aggregate unemployment rate (Figure 2.2b) clearly demonstrates that the number of unemployed job searchers rises during economic downturns.

The flow of workers from unemployment into employment, as displayed in Figure (2.2a), represents all unemployed job searchers that succeed in *meeting* an employer at the end of period $t - 1$. Since the reservation productivity at time t is still unknown, unemployed job searchers accept matches of any quality for the moment. Whether meeting an employer actually results in finding an active match in period t , depends on the realization of aggregate productivity. Therefore, Figure (2.2c) gives the worker's expected *meeting* rate, represented by a blue line, and the job *finding* rate, represented by a pink line. We observe that both transition rates are pro-cyclical. This is due to the fact that more unemployed job searchers compete for fewer vacancies during economic downturns. Moreover, the counter-cyclical flow in Figure (2.2a) suggests that the percentage rise in the number of unemployed job searchers is larger than the percentage fall in the worker's meeting rate.

Turning to the set of unmatched firms that have posted a vacancy (Figure 2.2e), we note that there are three different transition rates. The firms' *meeting* rate, represented by a blue line, gives the rate at which a firm can expect to meet a worker at the end of period $t - 1$. As some employed job searchers will immediately *reject* the offer (Figure 2.2f), the pink line gives the rate at which a firm can expect to be matched with a worker at the beginning of period t . In addition to that, some matches will be separated in the beginning of period t , due to exogenous and endogenous job *separation*. Hence, the yellow

⁸All graphs in Figure (2.2) represent stationary values, given different values of aggregate productivity. Consequently, the flow from employment to unemployment is always identical to the flow into the opposite direction.

⁹As the graphs depict stationary values, all endogenous separations at time t are due to newly formed matches in period $t - 1$ that do not become active in period t . In addition to that, we observe a spike in the endogenous separation rate every time the level of aggregate productivity decreases.

line shows at which rate a firm can actually expect to find an active job match in period t .

Finally, note that the introduction of variable on-the-job search uncouples aggregate search effort from the number of unemployed job searchers (Figure 2.1d). For this reason, the ratio of vacancies to the number of unemployed job searchers is no longer identical to “labor market tightness”, i.e. the ratio of vacancies to total search effort. Since on-the-job search is pro-cyclical, we observe that total search effort, i.e. the sum of all effort undertaken by employed and unemployed job searchers, is relatively stable over the business cycle. This stimulates firms to open more vacancies when aggregate productivity is high (Figure 2.2h). Consequently, we observe that the ratio of vacancies to unemployed job searchers is clearly pro-cyclical (Figure 2.2g). Labor market tightness, instead, is much less elastic. This indicates that the negative feedback effect of labor market tightness on vacancy creation, which prevails in the standard job matching model, is much weaker in a model with variable on-the-job search.

In summary, we notice that our model replicates salient features of the US labor market. Gross worker flows between employment and unemployment are counter-cyclical. Vacancies are pro-cyclical, while the number of unemployed job searchers is counter-cyclical. Hence, the model generated Beveridge curve is positively sloped.

2.3.4 Business Cycle Analysis

This section examines the quantitative performance of the job matching model with variable on-the-job search. We evaluate the model against business cycle moments of the US labor market from 1955:1 to 2008:4 (Table 2.2a). All data are logged and de-trended using a Hodrick and Prescott (1997) filter with smoothing parameter 1600. As a measure of aggregate activity we choose output per worker. We observe that both vacancies and unemployment are very volatile and very persistent. Vacancies are pro-cyclical, but the unemployment rate is counter-cyclical. The average real wage rate is less volatile than output per worker. In addition, we analyze whether the model is able to replicate the pattern of a relative stable *total* separation rate (Hall, 2005), which is due to the offsetting behavior of its components (Nagypál, 2008). In particular, the employment-to-unemployment (EU) transition rate is counter-cyclical, the employment-to-employment (EE) transition rate is pro-cyclical, and the employment to out-of-the labor force (EO) transition rate is essentially a-cyclical. Furthermore, Nagypál (2008) estimates that 50 to 60% of the volatility in the unemployment rate is due to composition changes in the total separation rate. The remaining share is caused by variations in the job finding rate.

As demonstrated by Shimer (2005), the baseline job matching model without on-the-job search is not able to replicate this pattern. The average real wage moves almost one-to-one with output per worker, providing firms not enough incentives to amplify the supply of vacancies over the business cycle. Hence, the model is not able to match the

high degree of labor market volatility found in the data (Table 2.2b). Moreover, we note that all model generated time series closely follow the exogenous stochastic process. This result suggests that the internal propagation mechanism of the matching model without physical capital is very weak. Besides, due to the exogenous separation margin, we observe that the model generated flow from employment to unemployment is, by construction, pro-cyclical (Davis, 2006). This implies that the baseline job matching model is not able to account for the composition changes in the total separation rate. Instead, all variation in the aggregate unemployment rate is attributed to the job creation margin.

Table 2.2c presents the second moments of our calibrated model. We observe that the inclusion of variable on-the-job search increases the volatility of vacancies and unemployment significantly. Compared to the baseline model, the volatility of unemployment rises by more than an order of a magnitude. This enormous rise is due to the counter-cyclical variations in the endogenous separation margin. Figure (2.3) illustrates that recessions involve a spike in the number of job separations. The unemployment rate tracks these movements almost exactly. On the other hand, the volatility of vacancies rises only modestly when variable on-the-job search is introduced. This result suggests that our model generates most of the volatility in unemployment along the job separation margin, and only little along the job creation margin.

This is a well-known problem in job matching models with endogenous separations (Mortensen and Nagypál, 2008), where counter-cyclical fluctuations in the job separation margin may induce strong counter-cyclical movements in the number of unemployed job searchers. Without on-the-job search, when aggregate search effort is directly linked to the number of unemployed job searchers, these movements stimulate firms to open more vacancies when aggregate productivity is low. In other words, the model generated Beveridge curve is positively sloped. This is in stark contrast to the data. Our model, however, uncouples total search effort from the number of unemployed job searchers. Since on-the-job search is pro-cyclical, aggregate search effort is relatively stable over the business cycle. Hence, consistent with the data, we observe that firms post more vacancies when aggregate productivity is high. Yet, the degree of amplification remains below the empirical estimate.

The main reason for the low volatility of vacancies is the fact that on-the-job search involves job offer rejections (Figure 2.2f). Firms suffer from job offer rejections by employed job searchers who prefer to stay in their old jobs. Each job offer rejection implies that the sunk cost of labor market search is lost. This effect discourages firms to open vacancies. Nevertheless, our model is able to generate a pro-cyclical time path of vacancies. This partial success is due to the impact of two effects. First, we observe that only employed job searchers in low productivity matches make great efforts to find a new employment opportunity. These workers try to “escape” imminent unemployment and, therefore, tend to accept most of the job offers. Second, conditional on job offer acceptance, firms en-

joy a higher expected payoff from an employed job searcher than from an unemployed. This is due to the fact that employed job searchers accept only attractive job offers and, therefore, are unlikely to quit later on. Hence, their expected match duration is longer (Nagypál, 2007).

Figure (2.4) presents the cyclical behavior of the *total* job separation rate and its components. The total job separation rate, represented by a yellow line, is made up of (i) the sum of the exogenous and the endogenous separation rate, represented by a pink line, and (ii) the employment-to-employment transition rate, represented by a blue line. The graph shows that the two main components of the total job separation rate are negatively correlated. The employment-to-employment transition rate is pro-cyclical, while the sum of the exogenous and the endogenous separation rate is counter-cyclical. The total job separation rate clearly exhibits less variability than its two main components.

Finally, we recall that our model is able to distinguish between the average real wage rate the real wage rate of an individual worker.¹⁰ The average real wage rate is clearly less volatile than the real wage rate of an individual worker. The co-movement with output, however, is almost the same between both variables.

2.4 Conclusion

This paper addresses the large degree of frictional wage dispersion found in US data. Hornstein et al. (2007) demonstrate that the average wage paid to observably identical workers is about 70% higher than the lowest wage in the sample. The standard job matching model without on-the-job search cannot replicate this pattern. With on-the-job search, however, unemployed job searchers are more willing to accept low wage offers since they can continue to seek for better employment opportunities. This explains why observably identical workers may be paid very different wages. Therefore, we examine the quantitative implications of variable on-the-job search (Nagypál, 2005) in a stochastic job matching model (Mortensen and Pissarides, 1994).

Our key result is that the inclusion of *variable* on-the-job search increases the degree of frictional wage dispersion by an order of a magnitude (from about 3% to 40%). Variable on-the-job search allows us to replicate the fact that search effort is most intense among workers in low-paid matches close to the separation margin (Fallick and Fleischman, 2004; Christensen et al., 2005). These “marginal” workers try to escape imminent unemployment and tend to accept most of the job offers (Nagypál, 2007). Hence, marginal workers enjoy very high employment-to-employment transition rates. These high career expectations stimulate unemployed job searchers to accept such low-paid matches. For this reason, we observe that the average wage paid to ex-ante identical workers in our model

¹⁰The analyzed individual real wage corresponds to an employed worker with idiosyncratic productivity level $a = 1.1$ who stays permanently on the same job.

is about 40% higher than the lowest wage in the sample.

Furthermore, we evaluate our modified job matching model at the business cycle frequencies. We observe that counter-cyclical variations in the endogenous job separation rate amplify the cyclical variations in the aggregate unemployment rate. Indeed, our model uncouples aggregate search effort from the number of unemployed job searchers. Since on-the-job search is pro-cyclical, we note that total search effort, i.e. the sum of all effort undertaken by employed and unemployed job searchers, is relatively stable over the business cycle. This stimulates firms to open more vacancies when aggregate productivity is high. As a consequence, we are able to replicate a negatively sloped Beveridge curve. This is an interesting result, given that models with endogenous separations, but without on-the-job search, imply that aggregate search effort is strongly counter-cyclical (Mortensen and Nagypál, 2008). Thus, in stark contrast to the data, firms are likely to post more vacancies when the number of unemployed job searchers is high.

It would be interesting to extend our analysis to a more general wage bargaining set up. To our knowledge, there is no study investigating all aspects of a multi-player bargaining game under variable on-the-job search. Cahuc et al. (2006) examine the implications of a multi-player bargaining game, where the search intensity of employed job searchers is constant (Burdett and Mortensen, 1998). They demonstrate that between-firm competition increases the average wage rate if employed job searchers can resume their old positions. Papp (2009) examines the implications of this approach in a general equilibrium model. On the other hand, Shimer (2006) analyzes a strategic bargaining game between an employed job searcher and a single firm, where the firm is willing to offer a higher wage rate in order to reduce the extent of transitions to other employers.

Appendix 2.A Tables

Parameter	Description	Value	Source
U	unemployment rate	0.10	Jones and Riddell (1999)
\tilde{f}	job finding rate	0.27	Hall (2005)
ρ^x	EU transition rate	0.01	Hall (2005)
ρ^n	EO transition rate	0.03	Hall (2005)
τ	EE transition rate	0.04	Hall (2005)
b	unemployment benefits	0.85	Costain and Reiter (2008)
ξ	firm's bargaining power	0.25	Yashiv (2000)
κ	vacancy posting cost	0.06	Yashiv (2000)
χ	matching function constant	0.45	normalization
γ	matching elasticity of vacancies	0.50	Petrongolo and Pissarides (2001)
Z	number of productivity states	29	
m	grid width	3	Tauchen (1986)
ϱ	1st order autocorrelation	0.97	Hagedorn and Manovskii (2008) ¹¹
σ_ϵ	standard deviation	0.007	Hagedorn and Manovskii (2008)

Table 2.1: Calibrated Parameter Values

	Y	V	U	V/U	W
$\sigma(X)$	0.01	0.13	0.11	0.24	0.01
$\sigma(X)/\sigma(Y/N)$	1	14.1	12.0	25.5	0.72
$\rho(X, Y/N)$	1	0.54	-0.42	0.49	0.38
$\rho(X_t, X_{t-1})$	0.71	0.90	0.89	0.90	0.70

(a) US Business Cycle Facts

	Y	V	U	V/U	W
$\sigma(X)$	0.02	0.02	0.01	0.03	0.02
$\sigma(X)/\sigma(Y/N)$	1	1.2	0.4	1.6	1
$\rho(X, Y/N)$	1	1	-0.95	1	0.84
$\rho(X_t, X_{t-1})$	0.78	0.75	0.80	0.78	0.70

(b) Second Moments without On-the-job Search

	Y	V	U	V/U	\bar{W}	W_i
$\sigma(X)$	0.01	0.04	0.06	0.11	0.01	0.01
$\sigma(X)/\sigma(Y/N)$	1	2.9	4.4	7.3	0.54	0.92
$\rho(X, Y/N)$	1	1	-0.99	0.99	0.97	1
$\rho(X_t, X_{t-1})$	0.77	0.77	0.76	0.77	0.76	0.77

(c) Second Moments with On-the-job Search

Table 2.2: US Business Cycle Facts and the Corresponding Moments of our Model Economy. All Data are logged and de-trended with an HP-Filter 1600. All US data (but the real wage) are taken directly from OECD.Stat. The real wage is constructed using the time series “Compensation of Employees: Wages and Salary Accruals” and “Total Nonfarm Payrolls: All Employees”.

¹¹See Footnote (7)

Appendix 2.B Figures

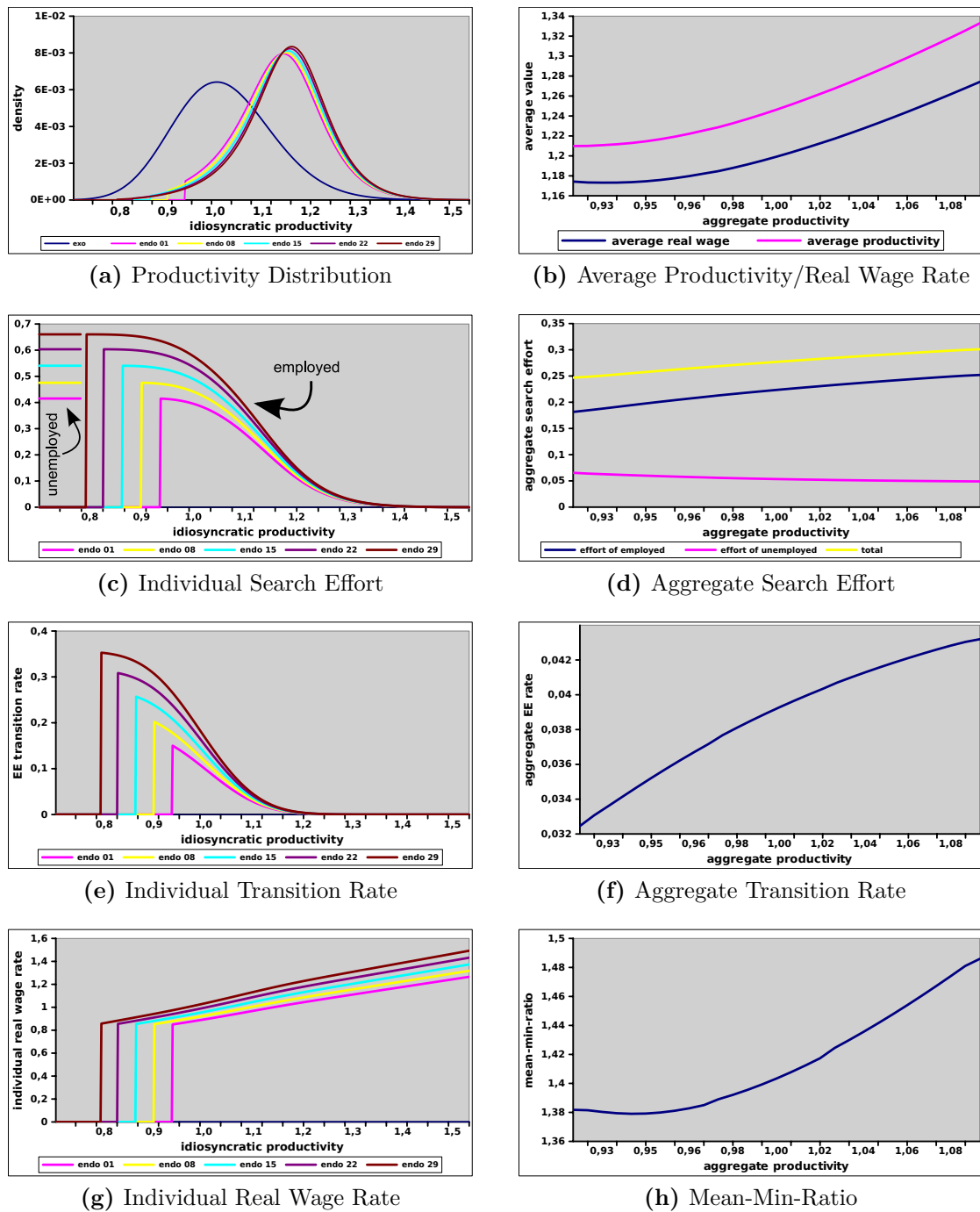


Figure 2.1: Endogenous Distribution, the Real Wage Rate, and On-the-job Search

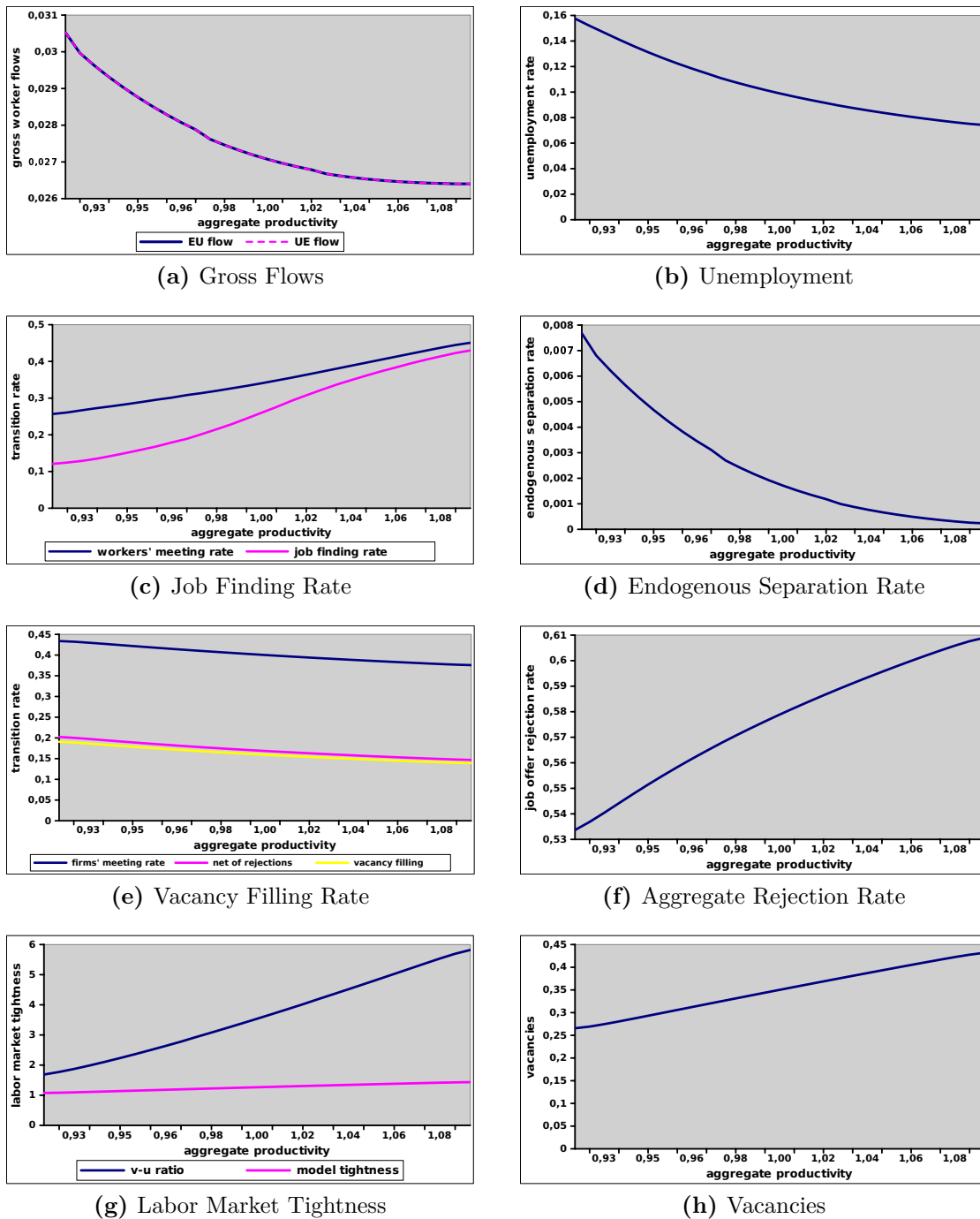


Figure 2.2: Aggregate Labor Market Dynamics

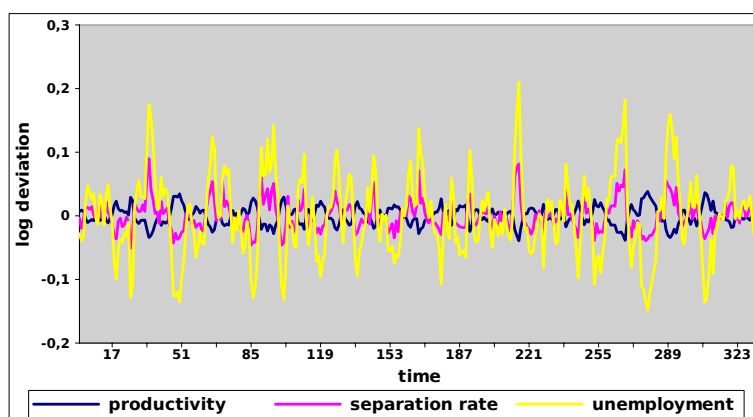


Figure 2.3: Counter-cyclical Spikes in the Separation Rate

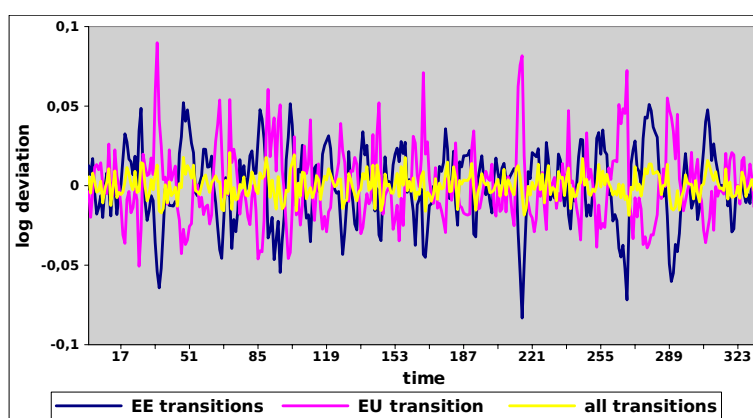


Figure 2.4: Composition Changes in the Total Separation Rate

Bibliography

- Angerhausen, J., Bayer, C. and Hehenkamp, B. (2009), ‘Strategic unemployment’, *Journal of Institutional and Theoretical Economics* . Forthcoming.
- Barlevy, G. (2002), ‘The sullyng effect of recessions’, *Review of Economic Studies* **69**(1), 65–96.
- Bowlus, A., Liu, H. and Robinson, C. (2002), ‘Business cycle models, aggregation, and real wage cyclicalty’, *Journal of Labor Economics* **20**(2), 308–335.
- Burdett, K. and Mortensen, D. T. (1998), ‘Wage differentials, employer size, and unemployment’, *International Economic Review* **39**(2), 257–73.
- Caballero, R. J. and Hammour, M. L. (1994), ‘The cleansing effect of recessions’, *American Economic Review* **84**(5), 1350–1368.
- Cahuc, P., Postel-Vinay, F. and Robin, J.-M. (2006), ‘Wage bargaining with on-the-job search: Theory and evidence’, *Econometrica* **74**(2), 323–364.
- Christensen, B. J., Lentz, R., Mortensen, D. T., Neumann, G. R. and Werwatz, A. (2005), ‘On-the-job search and the wage distribution’, *Journal of Labor Economics* **23**(1), 31–58.
- Costain, J. S. and Reiter, M. (2008), ‘Business cycles, unemployment insurance, and the calibration of matching models’, *Journal of Economic Dynamics and Control* **32**(4), 1120–1155.
- Davis, S. J. (2006), Comments on “Job loss, job finding, and unemployment in the U.S. economy over the past fifty years” by Robert E. Hall, *in* M. Gertler and K. Rogoff, eds, ‘NBER Macroeconomics Annual 2005’, MIT Press, Cambridge, Massachusetts, pp. 139–157.
- Elsby, M. W. L., Michaels, R. and Solon, G. (2009), ‘The ins and outs of cyclical unemployment’, *American Economic Journal: Macroeconomics* **1**(1), 84–110.
- Fahr, S. (2007), Essays on innovations, technology, and the labor market, PhD thesis, European University Institute. Chapter 1: On-the-job search and v-u volatility.
- Fallick, B. and Fleischman, C. A. (2004), Employer-to-employer flows in the U.S. labor market: The complete picture of gross worker flows, Finance and Economics Discussion Series No. 2004-34, Board of Governors of the Federal Reserve System (U.S.).
- Fujita, S. (2009), ‘Dynamics of worker flows and vacancies: Evidence from the sign restriction approach’, *Journal of Applied Econometrics* . Forthcoming.
- Hagedorn, M. and Manovskii, I. (2008), ‘The cyclical behavior of equilibrium unemployment and vacancies revisited’, *American Economic Review* **98**(4), 1692–1706.
- Hall, R. E. (2005), Job loss, job finding, and unemployment in the U.S. economy over the past fifty years, NBER Working Paper Series No. 11678, National Bureau of Economic Research.

- Hall, R. E. (2009), ‘Reconciling cyclical movements in the marginal value of time and the marginal product of labor’, *Journal of Political Economy* **117**(2), 281–323.
- Hart, R. A. (2006), ‘Worker-job matches, job mobility and real wage cyclicality’, *Economica* **73**(290), 287–298.
- Hodrick, R. J. and Prescott, E. C. (1997), ‘Postwar U.S. business cycles: An empirical investigation’, *Journal of Money, Credit and Banking* **29**(1), 1–16.
- Hornstein, A., Krusell, P. and Violante, G. L. (2007), Frictional wage dispersion in search models: A quantitative assessment, NBER Working Paper Series No. 13674, National Bureau of Economic Research.
- Jones, S. R. G. and Riddell, W. C. (1999), ‘The measurement of unemployment: An empirical approach’, *Econometrica* **67**(1), 147–162.
- Krause, M. U. and Lubik, T. A. (2007), On-the-job search and the cyclical dynamics of the labor market, Working Paper Series No. 779, European Central Bank.
- Krusell, P. and Smith, A. A. (1998), ‘Income and wealth heterogeneity in the macroeconomy’, *Journal of Political Economy* **106**(5), 867–896.
- Kydland, F. E. and Prescott, E. C. (1982), ‘Time to build and aggregate fluctuations’, *Econometrica* **50**(6), 1345–1370.
- Menzio, G. and Shi, S. (2009), Block recursive equilibria for stochastic models of search on the job, Working Paper No. 347, University of Toronto, Department of Economics.
- Moen, E. R. (1997), ‘Competitive search equilibrium’, *Journal of Political Economy* **105**(2), 385–411.
- Mortensen, D. and Nagypál, É. (2008), ‘Labor-market volatility in matching models with endogenous separations’, *Scandinavian Journal of Economics* **109**(4), 645–665.
- Mortensen, D. T. and Pissarides, C. A. (1994), ‘Job creation and job destruction in the theory of unemployment’, *Review of Economic Studies* **61**(3), 397–415.
- Nagypál, É. (2005), On the extent of job-to-job transitions, Unpublished manuscript, Northwestern University.
- Nagypál, É. (2007), Labor-market fluctuations and on-the-job search, Unpublished manuscript, Northwestern University.
- Nagypál, É. (2008), Worker reallocation over the business cycle: The importance of employer-to employer transitions, Unpublished manuscript, Northwestern University.
- Nash, J. (1953), ‘Two-person cooperative games’, *Econometrica* **21**, 128–140.
- Papp, T. K. (2009), Explaining frictional wage dispersion, Unpublished manuscript, Princeton University, Department of Economics.
- Petrongolo, B. and Pissarides, C. A. (2001), ‘Looking into the black box: A survey of the matching function’, *Journal of Economic Literature* **39**(2), 390–431.

- Pissarides, C. A. (1985), ‘Short-run equilibrium dynamics of unemployment, vacancies and real wages’, *American Economic Review* **75**(4), 676–690.
- Pries, M. and Rogerson, R. (2005), ‘Hiring policies, labor market institutions, and labor market flows’, *Journal of Political Economy* **113**(4), 811–839.
- Shimer, R. (2005), ‘The cyclical behavior of equilibrium unemployment and vacancies’, *American Economic Review* **95**(1), 25–49.
- Shimer, R. (2006), ‘On-the-job search and strategic bargaining’, *European Economic Review* **50**(4), 811–830.
- Shimer, R. (2007), Reassessing the ins and outs of unemployment, NBER Working Paper Series No. 13421, National Bureau of Economic Research.
- Solon, G., Barsky, R. and Parker, J. A. (1994), ‘Measuring the cyclical behavior of real wages: How important is composition bias?’, *The Quarterly Journal of Economics* **109**(1), 1–25.
- Tasci, M. (2007), On-the-job search and labor market reallocation, Working Paper No. 0725, Federal Reserve Bank of Cleveland.
- Tauchen, G. (1986), ‘Finite state markov-chain approximations to univariate and vector autoregressions’, *Economics Letters* **20**(2), 177–181.
- Yashiv, E. (2000), ‘The determinants of equilibrium unemployment’, *American Economic Review* **90**(5), 1297–1322.
- Yashiv, E. (2008), ‘U.S. labor market dynamics revisited’, *Scandinavian Journal of Economics* **109**(4), 779–806.

Chapter 3

Shocks and Frictions under Right-to-manage Wage Bargaining: A Transatlantic Perspective

3.1 Introduction

This paper introduces staggered right-to-manage wage bargaining into a New Keynesian business cycle model. Our key result is that a reasonably calibrated version of the model is able to generate persistent responses in output, inflation, and total labor input to both neutral technology and monetary policy shocks. Furthermore, we compare the model's dynamic behavior when calibrated to the US and to a European economy. We find that the degree of price rigidity explains most of the differences in response to a monetary policy shock. Differences in the degree of wage rigidity, instead, alter the dynamics of the model economy only by little. When the economy is hit by a neutral technology shock, both price and wage rigidities turn out to be important. Apart from that, our results indicate that matching frictions matter primarily for the dynamics of the labor market.

We introduce frictional labor markets into a New Keynesian business cycle model akin to Christiano et al. (2005) and Smets and Wouters (2003). Households' preferences are represented by an additive utility function over consumption, working time, and real money holdings. The composite consumption good consists of a CES aggregate of differentiated intermediate goods. These goods are produced by monopolistically competitive intermediate good firms, facing Calvo (1983) type restrictions in price setting on the product market. Factor markets for capital and labor services, instead, are assumed to be perfectly competitive. Households accumulate physical capital and rent capital services at a variable utilization rate to the intermediate good firms. Labor services are provided by hiring firms searching for workers on frictional labor markets (Christoffel and Kuester, 2008). Upon matching, firm-worker pairs first bargain over the real wage rate which is subject to staggered wage contracts. In the second step, hiring firms may choose the number of hours per worker unilaterally. In this setting, which is referred to as “right-to-

manage” wage bargaining (Trigari, 2006), the real wage rate is allocative for the number of hours per worker. Consequently, any rigidity in the real wage rate is transmitted via the New Keynesian Phillips Curve into persistent movements of inflation. This feature of right-to-manage wage bargaining is referred to as the “wage channel”.

We then examine the effects of two structural shocks. The first shock represents a sudden increase in the short term nominal interest rate. Using different identification strategies and data sets, Sims (1992), Leeper et al. (1996), and Christiano et al. (1999, 2005), among others, demonstrate that such a shock leads to distinct U-shaped responses in both output and inflation. Moreover, Ravn and Simonelli (2008) show that also the dynamic time path of total labor input follows a U-shaped pattern in the aftermath of a monetary policy shock.

Second, we examine the impact of a neutral technology shock. Evidence on the effects of technology shocks is rather controversial. As shown by Galí (1999), a positive technology shock generates a persistent rise in output and a persistent decline in the inflation rate. In addition to that, he finds a *negative* correlation between technology and total labor input. The latter observation is in stark contrast to the predictions of the baseline RBC model and, thus, has sparked an intense and still ongoing debate in the literature. While Francis and Ramey (2005) provide evidence in favor of his result, Christiano et al. (2003, 2004) and Uhlig (2004) question its robustness.¹ The study by Ravn and Simonelli (2008) estimates a SVAR model of the US labor market which includes 4 different shocks: A neutral technology shock, an investment specific technology shock, a monetary policy shock, and a government spending shock. They argue that the large set of identified shocks minimizes the problem of mis-specification and, therefore, yields more robust results. Their findings confirm the conventional wisdom that a neutral technology shock leads to a positive and hump-shaped response in output and a negative and U-shaped response in inflation. Furthermore, they provide robust evidence that (i) output and total labor input are *positively* correlated at the business cycle frequencies in response to a neutral technology shock and that (ii) the impact response of the employment level is positive. The impact response of total labor input, however, depends on the question whether hours per worker are level or difference stationary.²

When we calibrate the model to the US economy, we observe that it is able to generate persistent output responses to monetary policy shocks. This seems to be the main contribution of our paper. New Keynesian models with Nash bargaining (e.g. Walsh, 2005), instead, are not able to replicate this pattern once capital accumulation is introduced (Heer and Maussner, 2007). This effect is due to the alternative bargaining approach.

¹As pointed out by Peersman and Straub (2005) and Heer and Maussner (2007), the impact of technology on total labor input depends crucially on the question whether hours per worker are level or difference stationary.

²Their results are broadly consistent with the estimates of Braun et al. (2006), who use an alternative identification strategy.

Right-to-manage wage bargaining establishes a direct link between the real wage rate and real marginal costs. Hence, any rigidity in the average real wage rate dampens the response in real marginal costs. This so-called “wage channel” has two important implications. First, the reduced elasticity of real marginal costs is transmitted via the New Keynesian Phillips Curve into persistent movements of inflation. Second, we note that the sluggish response in real marginal costs dampens additionally the response of the real interest rate. In the case of variable capital utilization, this leads to a hump-shaped decline in the input of capital services. Consequently, given that matching frictions induce a sluggish response in total labor input, we note that the response of aggregate output reaches its minimum not impact, but just in the second period after an innovation in monetary policy.

In response to a neutral technology shock, our model is able to replicate a hump-shaped response in output and a U-shaped response in inflation. Turning to the labor market, we observe that unemployment exhibits a negative impact response and then continues to decrease for about 6 quarters. Hours per worker, instead, fall on impact, but then rise for about 2 years before eventually falling. Hence, consistent with the findings of Ravn and Simonelli (2008), we observe a positive correlation between output and total labor input at the business cycle frequencies in response to a neutral technology shock.

Apart from that, we calibrate our model to a European economy and compare its dynamic behavior with the US model economy. The European model economy differs in terms of a greater price rigidity parameter, a greater real wage rigidity parameter, and a larger degree of matching frictions in the labor market. In particular, we account for the fact that European transition rates between employment and unemployment are considerably lower. The higher value of the average European unemployment rate is mainly due to a more generous replacement ratio.

In response to a monetary policy shock, we observe that the decline in output and total labor input is larger and more protracted in the European model economy. The impulse response of inflation, however, shows a smaller impact response and a more persistent adjustment path. These three observations can be attributed to the greater price rigidity parameter. Further, the impulse response of the European unemployment rate exhibits a clear hump-shape. In the US model economy, on the contrary, the unemployment rate spikes on impact and then converges quickly to its steady state value. This pattern is mainly explained by the smaller value of the job separation rate which delays labor market turnover.

When the two model economies are hit by a neutral technology shock, we observe more interdependencies between the three frictions considered. On the one hand, the larger degree of price rigidity raises the amplitude of output and inflation. On the other hand, the larger degree of real wage rigidity dampens the amplitude and delays the speed of convergence. In total, the amplitude of both impulse responses remains almost con-

stant, but convergence is slower under the European calibration. Again, the labor market calibration affects primarily the response of the unemployment rate. First, we note that the percentage impact response of the European unemployment rate is only about 1/4 of the US value. Second, in the same way as above, greater price rigidity increases the amplitude of the unemployment rate, while a large degree of real wage rigidity dampens the fluctuations. As a result, the joint impact of the two Calvo type rigidities raises the persistence of the unemployment rate, but leave its amplitude virtually unchanged.

The remainder of this paper is organized as follows. Section 3.2 presents the model environment. Section 3.3 calibrates the model and evaluates its quantitative performance. We investigate the mechanism of the right-to-manage bargaining model based on a calibration to the US economy. In addition, we examine the differences between the US and a European model economy. Section 3.4 concludes.

3.2 The Model Environment

3.2.1 Labor Market Frictions

Labor market frictions are represented by a Cobb-Douglas matching function that relates aggregate job matches M_t to the number of vacancies that are posted by the firms V_t and the number of unemployed job searchers U_{t-1} .³

$$M_t(V_t, U_{t-1}) = \chi V_t^\mu U_{t-1}^{1-\mu} \leq \min[V_t, U_{t-1}]. \quad (3.1)$$

The ratio between vacancies and unemployed job searchers ($V_t/U_{t-1} = \theta_t$) measures the tightness of the labor market. By linear homogeneity of the matching function, the vacancy filling rate $q(\theta_t)$ and the job finding rate $q(\theta_t)\theta_t$ depend solely on the value of labor market tightness:

$$q(\theta_t) \equiv \frac{M_t}{V_t} = \chi \left(\frac{U_{t-1}}{V_t} \right)^{1-\mu}, \quad q(\theta_t)\theta_t \equiv \frac{M_t}{U_{t-1}} = \chi \left(\frac{V_t}{U_{t-1}} \right)^\mu. \quad (3.2)$$

The tighter the labor market, the longer the expected time to fill a vacancy, but the shorter the expected search for a job (and vice versa). The fact that firms and households do not internalize these adverse effects on the aggregate return rates gives rise to congestion externalities.

At the end of each period, new job matches are formed and a fraction of pre-existing jobs is terminated. Consistent with the results of Shimer (2007), we assume a constant job destruction rate ρ . Hence, the law of motion for the aggregate level of employment is

³We follow the textbook job matching model (Pissarides, 1985, 2000) that abstracts from movements into and out of the labor force. Hence, $U_t = 1 - N_t$ holds.

given by:

$$N_t = (1 - \rho)N_{t-1} + M_t. \quad (3.3)$$

Moreover, we assume that the real wage rate is subject to staggered wage contracts (Calvo, 1983). This implies that — new and ongoing — firm-worker pairs are able to bargain over the real wage rate w_t^* only with probability $(1 - \omega_w)$. Otherwise, the real wage rate in ongoing firm-worker pairs remains constant. New firm-worker pairs that are unable to negotiate simply adopt the average real wage rate of the previous period w_{t-1} .⁴ Hence, the evolution of the average real wage rate w_t is governed by following law of motion:

$$w_t = \omega_w w_{t-1} + (1 - \omega_w)w_t^*. \quad (3.4)$$

3.2.2 Households

There is a large number of households, each of which consists of a continuum of individuals. Household members derive utility from the composite consumption good $C_{j,t}$ and real money holdings $(M/P)_{j,t}$. Employed household members additionally suffer disutility from working time $h_{j,t}$. Hence, preferences of an individual household member j are given as:

$$U(C_{j,t}, (M/P)_{j,t}, h_{j,t}) = \frac{(C_{j,t} - \psi_c C_{j,t-1})^{1-\sigma_c}}{1-\sigma_c} + \psi_q \frac{(M/P)_{j,t}^{1-\sigma_q}}{1-\sigma_q} - \psi_f \left(\frac{h_{j,t}^{1+\sigma_f}}{1+\sigma_f} \right), \quad (3.5)$$

where ψ_c measures the degree of habit persistence in consumption and $1/\sigma_f$ denotes the elasticity of intertemporal substitution in the supply of hours worked.

Employed and unemployed household members insure each other completely against idiosyncratic income risk from unemployment (Merz, 1995; Andolfatto, 1996). Thus, the budget constraint of the representative household can be written as:

$$\begin{aligned} C_t + I_t + (M/P)_t + B_t + a(x_t) \bar{k}_{t-1} &= \int_0^{N_{t-1}} w_{j,t} h_{j,t} dj + bU_{t-1} + \Pi_t + r_t K_t \\ &+ \frac{(M/P)_{t-1}}{\pi_t} + \frac{R_{t-1}}{\pi_t} B_{t-1} - T_t. \end{aligned} \quad (3.6)$$

Employed household members earn the real wage rate $w_{j,t}$ per working hour $h_{j,t}$, while unemployed household members U_{t-1} receive unemployment benefits b . The lump-sum transfer T_t imposed by the government finances unemployment benefits, governmental consumption, and rebates any seigniorage revenue to the households (see section 3.2.6). Government bonds B_t pay a nominal interest rate R_t in period $t+1$. Moreover, households

⁴As demonstrated by Haefke et al. (2009) and Pissarides (2009), wages in new hires are significantly more volatile than wages in incumbent matches. Under right-to-manage wage bargaining, however, accounting for this aspect hardly changes the quantitative results (Christoffel et al., 2009). For this reason, we choose the described set-up for analytical convenience.

receive lump-sum dividends⁵ Π_t remitted by firms and capital income $r_t K_t$. Effective capital services K_t are given by the physical capital stock \bar{K}_{t-1} times the capital utilization rate x_t . Following (Schmitt-Grohe and Uribe, 2004), the costs of variations in the degree of capital utilization are given as:

$$a(x_t) = \frac{\chi_a}{2} (x_t - 1)^2 + \chi_b (x_t - 1). \quad (3.7)$$

Hence, provided that the steady state value of capital utilization is normalized to unity, the steady state of the model economy will be independent of $a(x_t)$, i.e. $a(1) = 0$. Nevertheless, capital adjustment costs affect the utilization elasticity with respect to the rental rate of capital: $[a'(x)/(a''(x) x)]|_{x=1} = \chi_b/\chi_a$.

Furthermore, the law of motion for the physical capital stock is given by:

$$\bar{K}_t = (1 - \delta) \bar{K}_{t-1} + \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t, \quad (3.8)$$

$$\text{where } S(\cdot) = \frac{\chi_s}{2} \left(\frac{I_t - I_{t-1}}{I_{t-1}}\right)^2, \quad (3.9)$$

is restricted to satisfy $S(1) = S'(1) = 0$ and $S''(1) = \chi_s > 0$. The law of motion for the household's employment share reads as follows:

$$N_t = (1 - \rho) N_{t-1} + q_t \theta_t U_{t-1}. \quad (3.10)$$

First Order Conditions

Provided stochastic time paths for $\{R_t, r_t, \Pi_t, \pi_t, \theta_t, T_t, \tilde{h}_t | \geq 0\}$, and a set of initial conditions for the state variables $\{\bar{K}_0, N_0, \tilde{w}_0\}$, the representative household chooses contingency plans $\{C_t, x_t, B_t, M_t, I_t, \bar{K}_t | t \geq 0\}$ that maximize its expected discounted utility:⁶

$$\mathcal{U}_t(\bar{K}_{t-1}, N_{t-1}, \tilde{w}_t) = \max_{C_t, x_t, B_t, M_t, I_t, \bar{K}_t} \left\{ U(C_t, (M/P)_t, \tilde{h}_t) + \beta \mathcal{U}_{t+1}(\bar{K}_t, N_t, \tilde{w}_{t+1}) \right\} \quad (3.11)$$

⁵Aggregate dividends $\Pi_t = \Pi_y + \Pi_n$ are given as the sum of dividends remitted by intermediate good firms and hiring firms, respectively.

⁶The distribution of real wages and hours over matched firm-worker pairs is denoted by \tilde{w}_t and \tilde{h}_t , respectively.

These choices have to satisfy following first order conditions:

$$\lambda_{c,t} = (C_t - \psi_c C_{t-1})^{-\sigma_c} - \beta E_t [\psi_c (C_{t+1} - \psi_c C_t)^{-\sigma_c}], \quad (3.12)$$

$$r_t = a'(x_t) = \chi_a(x_t - 1) + \chi_b, \quad (3.13)$$

$$\lambda_{c,t} = \beta E_t \left[\frac{\lambda_{c,t+1}}{\pi_{t+1}} \right] R_t, \quad (3.14)$$

$$\lambda_{c,t} = [\psi_q (M/P)_t^{-\sigma_q}] + \beta E_t \left[\frac{\lambda_{c,t+1}}{\pi_{t+1}} \right] \Leftrightarrow \left(\frac{M}{P} \right)^{\sigma_q} = \frac{\psi_q}{\lambda_{c,t}} \frac{R_t}{R_t - 1}, \quad (3.15)$$

$$\begin{aligned} \lambda_{c,t} = & \lambda_{k,t} \left[\left(1 - S \left(\frac{I_t}{I_{t-1}} \right) \right) - \left(\frac{I_t}{I_{t-1}} \right) \chi_s \left(\frac{I_t - I_{t-1}}{I_{t-1}} \right) \right] \\ & + \beta E_t \left[\lambda_{k,t+1} \chi_s \left(\frac{I_{t+1} - I_t}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right], \end{aligned} \quad (3.16)$$

$$Q_t = \frac{\lambda_{k,t}}{\lambda_{c,t}} = \beta E_t \left\{ \left(\frac{\lambda_{c,t+1}}{\lambda_{c,t}} \right) (Q_{t+1}(1 - \delta) - a(x_{t+1}) + r_{t+1}x_{t+1}) \right\}. \quad (3.17)$$

The first order conditions describe the marginal utility of consumption (3.12), the relation between the rental rate of capital r and the utilization rate x_t (3.13), the Euler equation for government bonds (3.14), the demand for real money holdings (3.15), optimal investment (3.16), and the real value of physical capital (3.17).

The Net Benefit of Additional Employment

The net marginal benefit to the household when an unemployed household member finds a job is given by the following expression:

$$\begin{aligned} \mathcal{W}_t(w_{j,t}) = \left(\frac{\partial \mathcal{U}_t}{\partial N_{t-1}} \right) - \left(\frac{\partial \mathcal{U}_t}{\partial U_{t-1}} \right) = & \lambda_{c,t} [w_{j,t} h_{j,t} - b] - \psi_f (h_{j,t}^{1+\sigma_f} / 1 + \sigma_f) \\ & + \omega_w \beta E_t [(1 - \rho) \mathcal{W}(w_{j,t}) - q(\theta_t) \theta_t \mathcal{W}(w_t)] \\ & + (1 - \omega_w) (1 - \rho - q(\theta_t) \theta_t) \beta E_t [\mathcal{W}_2(w_{j,t+1}^*)]. \end{aligned} \quad (3.18)$$

One additional employed household member increases the net income of the household, but suffers disutility from working time. Besides that, the household gains the continuation value of the current real wage rate $\mathcal{W}(w_{j,t})$ with probability $(1 - \rho)\omega_w$, the continuation value of the re-negotiated real wage rate $\mathcal{W}(w_{j,t+1}^*)$ with probability $(1 - \rho)(1 - \omega_w)$, and loses the continuation value of unemployment. The latter is determined by the job finding rate $q(\theta_t)\theta_t$, the expected value of a job that pays the average real wage rate $\mathcal{W}(w_t)$, and the expected value of a job that pays the re-negotiated real wage rate $\mathcal{W}(w_{j,t+1}^*)$.

3.2.3 The Composite Consumption Good

The composite consumption good consists of a CES aggregate of differentiated intermediate goods:

$$C_t = \left[\int_0^1 C_{it}^{(\xi_p-1)/\xi_p} di \right]^{\xi_p/(\xi_p-1)}, \quad (3.19)$$

where $\xi_p > 1$ is the elasticity of substitution between differentiated intermediate goods C_{it} . Given that P_{it} denotes the price for intermediate good i , equation (3.19) implies that its relative demand is given as:

$$\frac{C_{i,t}}{C_t} = \left(\frac{P_{it}}{P_t} \right)^{-\xi_p}. \quad (3.20)$$

Integrating (3.20) and imposing (3.19), we obtain the associated minimum expenditure price index:

$$P_t = \left[\int_0^1 P_{i,t}^{1-\xi_p} di \right]^{1/(1-\xi_p)}. \quad (3.21)$$

3.2.4 Intermediate Good Firms

Each intermediate good $i \in [0, 1]$ is produced by a single firm and sold in a market characterized by monopolistic competition.⁷ The productive process in this sector can be described by a Cobb-Douglas production function with constant returns to scale:

$$Y(K_t, L_t) = \epsilon_t^z K_t^\alpha L_t^{1-\alpha}, \quad (3.22)$$

where ϵ_t^z represents total factor productivity subject to an exogenous shock specified by the following autoregressive process:

$$\epsilon_t^z = \rho_z \epsilon_{t-1}^z + \iota_t^z, \text{ where } \iota_t^z \sim N(0, \sigma_{\iota^z}^2). \quad (3.23)$$

Following de Walque et al. (2008), we assume perfect competition on the factor markets. Intermediate good firms rent capital services K_t directly from the households and labor services L_t from hiring firms. Constant returns to scale in production in combination with price-taking behavior on the factor markets yield following factor prices for capital (r_t) and labor services (W_t), respectively:

$$r_t = \lambda_{y,t} Y_1(K_t, L_t), \quad (3.24)$$

$$W_t = \lambda_{y,t} Y_2(K_t, L_t). \quad (3.25)$$

This implies that the real marginal cost $\lambda_{y,t}$ can be written as:

$$\lambda_{y,t} = \frac{1}{\epsilon_t^z} \left(\frac{r_t}{\alpha} \right)^\alpha \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha}. \quad (3.26)$$

⁷Given symmetry, we will drop the subscript i in the following.

On the product market, intermediate good firms face Calvo (1983) type restrictions in price setting. In the beginning of period t , only a fraction $1 - \omega_p$ of intermediate good firms is able to re-optimize the price of its variety. Intermediate good firms that cannot re-optimize simply index their prices to lagged inflation π_{t-1} . This specification yields following log-linearized New Keynesian Phillips Curve:

$$\hat{\pi}_t = \frac{1}{1 + \beta} \hat{\pi}_{t-1} + E_t \left[\frac{\beta}{1 + \beta} \hat{\pi}_{t+1} \right] + \frac{(1 - \omega_p)(1 - \beta\omega_p)}{\omega_p(1 + \beta)} \hat{\lambda}_{y,t}. \quad (3.27)$$

3.2.5 Employment relations

Hiring Firms

Labor services are provided by specialized hiring firms (Christoffel and Kuester, 2008). There is a continuum of *potential* hiring firms on the unit interval. Each hiring firm can hire at most one worker j . Hiring firms with filled positions N_{t-1} produce labor services according to a decreasing returns to scale technology $H(h_{j,t}) = h_{j,t}^{\sigma_h}$, with $\sigma_h < 1$. Hence, the units of aggregate labor services L_t produced in period t are given by:

$$L_t = N_{t-1} H(\tilde{h}_t) = \int_0^{N_{t-1}} h_{j,t}^{\sigma_h} dj = \int_0^1 L_{i,t} di \quad (3.28)$$

The hiring firm j rents the amount $H(h_{j,t})$ of labor services to intermediate good firms at rate W_t on a competitive market. The worker receives the real wage rate $w_{j,t}$ per hour worked $h_{j,t}$.

If the match survives exogenous job destruction at the end of period t , the firm and the worker may re-negotiate the real wage rate with probability $(1 - \omega_w)$ in period $t + 1$. Otherwise, the real wage rate remains constant. Hence, the value of a filled position to the hiring firm reads as:

$$\mathcal{J}_t(w_{j,t}) = W_t H(h_{j,t}) - w_{j,t} h_{j,t} + (1 - \rho) \beta E_t \left[\left(\frac{\lambda_{c,t+1}}{\lambda_{c,t}} \right) (\omega_w \mathcal{J}_{t+1}(w_{j,t}) + (1 - \omega_w) \mathcal{J}_{t+1}(w_{t+1}^*)) \right]. \quad (3.29)$$

Hiring firms with unfilled positions may decide whether or not to open a vacancy. Posting a vacancy entails a cost κ per period. Therefore, the hiring firm can expect to gain the value of a filled position \mathcal{J}_{t+1} with probability $q(\theta_t)$ in the next period. With probability $1 - q(\theta_t)$ the vacancy remains unfilled. Upon matching, the firm-worker pair j will be able to bargain over the real wage rate w_{t+1}^* with probability $(1 - \omega_w)$. If the hiring firm and the worker are unable to bargain, they will adopt the average real wage rate of the previous period, i.e. w_t . Thus, the value of an unfilled vacancy \mathcal{V}_t is given as:

$$\mathcal{V}_t = -\kappa + \beta E_t \left[\left(\frac{\lambda_{c,t+1}}{\lambda_{c,t}} \right) \left(q(\theta_t) \{ \omega_w \mathcal{J}_{t+1}(w_t) + (1 - \omega_w) \mathcal{J}_{t+1}(w_{t+1}^*) \} + [1 - q(\theta_t)] \mathcal{V}_{t+1} \right) \right]. \quad (3.30)$$

Free entry into the matching market ensures that the hiring firm's outside option, i.e. the value of an unfilled vacancy, is zero in each period: $\mathcal{V}_t = 0 \forall t$. Hence, the non-arbitrage condition for vacancy creation is given by:

$$\frac{\kappa}{q(\theta_t)} = \beta E_t \left[\left(\frac{\lambda_{c,t+1}}{\lambda_{c,t}} \right) \left[\omega_w \mathcal{J}_{t+1}(w_t) + (1 - \omega_w) \mathcal{J}_{t+1}(w_{t+1}^*) \right] \right]. \quad (3.31)$$

Right-to-Manage Wage Bargaining

Right-to-manage wage bargaining (Trigari, 2006), in contrast, presumes following sequential setting: First, both parties agree on a real wage rate w_t according to the Nash rule. Second, the hiring firm may choose the number of hours per worker $h_{j,t}$ unilaterally. Thus, the hiring firm sets hours per worker in order to maximize \mathcal{J}_t :

$$w_{j,t} = \sigma_h W_t \frac{H(h_{j,t})}{h_{j,t}} \quad (3.32)$$

$$\Leftrightarrow h_t(w_{j,t}) = \left(\sigma_h \frac{W_t}{w_{j,t}} \right)^{1/(1-\sigma_h)} \quad (3.33)$$

The first order condition (3.32) states that hiring firms set hours per worker such that the real wage rate equals the marginal product per hour worked. Provided that σ_h is close to one, this implies that movements in the average real wage rate w_t translate almost one-to-one into changes in the competitive price of labor services W_t and, thus, into real marginal costs $\lambda_{y,t}$. This feature of the right-to-manage bargaining model is referred to as the “wage channel”.

Furthermore, equation (3.33) points out that hours per worker under right-to-manage are a function of the real wage rate. During the wage bargaining, both parties internalize the impact of the real wage rate on the number of hours per worker. Hence, the maximization of the Nash product yields following sharing rule:

$$\eta \left(\frac{\partial (\mathcal{W}_t(w_{j,t})/\lambda_{c,t})}{\partial w_{j,t}} \right) \Big|_{\mathcal{J}_t^*} = (1 - \eta) \left(- \frac{\partial \mathcal{J}_t(w_{j,t})}{\partial w_{j,t}} \right) \Big|_{(\mathcal{W}_t^*/\lambda_{c,t})}, \quad (3.34)$$

where $0 < \eta < 1$ denotes the relative (“nominal”) bargaining power of the household. The net marginal benefit of an increase in the real wage rate to the worker, and the loss

to the hiring firm, respectively, are given as:⁸

$$\begin{aligned}
\delta_t^W &= \left(\frac{\partial (\mathcal{W}_t(w_{j,t})/\lambda_{c,t})}{\partial w_{j,t}} \right) \Big| w_t^* \\
&= \frac{w_t^* h_t^*}{1 - \sigma_h} \left[\frac{1}{w_t^*} \left(\psi_f \frac{(h_t^*)^{\sigma_f}}{\lambda_{c,t}} \right) - \sigma_h \right] + (1 - \rho) \beta \omega_w E_t \left[\left(\frac{\lambda_{c,t+1}}{\lambda_{c,t}} \right) \left(\frac{w_{t+1}^*}{w_t^*} \right)^{(1+\sigma_f)/(1-\sigma_h)} \right. \\
&\quad \left. w_{t+1}^* \sum_{k=1}^{\infty} \frac{1}{1 - \sigma_h} \left(\sigma_h \frac{W_{t+k}}{w_{j,t+k}} \right)^{1/(1-\sigma_h)} \left(\frac{1}{w_{j,t+k}} \right) \left(\frac{\psi_f}{\lambda_{c,t}} \right) \left(\sigma_h \frac{W_{t+k}}{w_{j,t+k}} \right)^{\psi_f/(1-\sigma_h)} \right] \\
&\quad - (1 - \rho) \beta \omega_w E_t \left[\left(\frac{\lambda_{c,t+1}}{\lambda_{c,t}} \right) \left(\frac{w_{t+1}^*}{w_t^*} \right)^{\sigma_h/(1-\sigma_h)} w_{t+1}^* \sum_{k=1}^{\infty} \frac{\sigma_h}{1 - \sigma_h} \left(\sigma_h \frac{W_{t+k}}{w_{j,t+k}} \right)^{1/(1-\sigma_h)} \right] \\
\delta_t^F &= \left(-\frac{\partial J_t(w_{i,t})}{\partial w_{i,t}} \right) \Big| w_t^* \\
&= w_t^* h_t^* + (1 - \rho) \beta \omega_w E_t \left[\left(\frac{\lambda_{c,t+1}}{\lambda_{c,t}} \right) \left(\frac{w_{t+1}^*}{w_t^*} \right)^{\sigma_h/(1-\sigma_h)} \right].
\end{aligned} \tag{3.35}$$

$$\begin{aligned}
&= w_t^* h_t^* + (1 - \rho) \beta \omega_w E_t \left[\left(\frac{\lambda_{c,t+1}}{\lambda_{c,t}} \right) \left(\frac{w_{t+1}^*}{w_t^*} \right)^{\sigma_h/(1-\sigma_h)} \right].
\end{aligned} \tag{3.36}$$

Thus, using equations (3.18), (3.25), (3.28), (3.29), and (3.34) the steady-state wage equation can be written as:

$$wh = \eta^* \left[\lambda_y (1 - \alpha) \frac{Y}{n} + \kappa \theta \right] + (1 - \eta^*) \left[b + \frac{\psi_f}{\lambda_c} \frac{h^{1+\sigma_f}}{1 + \sigma_f} \right], \tag{3.37}$$

where the effective bargaining weight η_t^* is a time-dependent variable:

$$\eta_t^* = \frac{\eta \delta_t^W}{\eta \delta_t^W + (1 - \eta) \delta_t^F}. \tag{3.38}$$

3.2.6 Government and Monetary Authority

The government finances unemployment benefits b , issues bonds B_t that pay a nominal interest rate R_t in period $t + 1$, and consumes a constant share of output $G_t = gY_t$. Any seigniorage revenue is rebated to the households. Each period, the budget balance is maintained by imposing a lump-sum tax T_t :

$$T_t + (M/P)_t + B_t = bU_{t-1} + \frac{(M/P)_{t-1}}{\pi_t} + B_{t-1} \frac{R_{t-1}}{\pi_t} + G_t. \tag{3.39}$$

Monetary policy obeys a generalized Taylor (1993) rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\phi_r} \left(\left(\frac{\pi_t}{\pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y} \right)^{(1-\phi_r)} \exp(\epsilon_t^r), \tag{3.40}$$

⁸We have multiplied both expressions with the re-negotiated real wage rate w_t^* .

where ϵ_t^r is a serially uncorrelated, mean zero stochastic process and $\phi_\pi > 1$. Accordingly, the monetary authority sets the short-term nominal interest rate depending on the lagged nominal interest rate R_{t-1} , current inflation π_t , and the current level of economic activity Y_t (Clarida et al., 2000).

3.2.7 Market Clearing

The model economy is closed by the resource constraint. It postulates that output is divided into private consumption, investment, government consumption, vacancy posting costs, and capital utilization costs.

$$Y_t = C_t + I_t + G_t + \kappa V_t + a(x_t) \bar{k}_{t-1}. \quad (3.41)$$

3.3 Model Evaluation

3.3.1 Calibration US

We analyze the cyclical behavior of the log-linearized model economy around the non-stochastic steady state. The parameters are chosen to be largely consistent with those standard in the literature. The time period of the model corresponds to one quarter.

Preferences The discount factor β is chosen to match an annual real interest rate of 4 percent (Kydland and Prescott, 1982). Following Christiano et al. (2005), we assume logarithmic preferences in consumption ($\sigma_c = 1$), together with external habit formation ($\psi_c = 0.65$). In addition, we borrow their estimates for the interest semi-elasticity of money demand (0.96, implying $\sigma_q = 6.3$) and the elasticity of substitution between differentiated intermediate goods ($\xi_p = 6$). The latter value implies that the average mark-up $((1/\lambda_y) - 1)$ is equal to 20%. For the intertemporal elasticity of labor supply ($1/\sigma_f$) we target a value that lies within the range $[0.3 - 0.7]$ estimated by MaCurdy (1983).

Production and the Capital Market The monthly depreciation rate δ is set to match an annual rate of 10% (Kydland and Prescott, 1982). In addition, we adopt following two parameters from Christiano et al. (2005): First, we set $\alpha = 0.36$ which corresponds to a steady state labor share slightly below 64%.⁹ Second, the scaling parameter of the investment adjustment cost function ($\chi_s = 2$) is chosen such that the elasticity of investment with respect to a one percent temporary increase in the current price of installed capital is equal to 0.4. Our chosen value for the elasticity of capital utilization with respect to the rental rate of capital $(\chi_b/\chi_a) = 1$ is close to the estimate by Smets and Wouters (2007).

⁹In labor search models the labor share is slightly lower than the production elasticity of labor.

Matching and the Labor Market Following Shimer (2005), we target an average unemployment rate $U = 5.7\%$ and a steady state job finding rate $q(\theta)\theta = 83.4\%$. This requires setting the job destruction rate ρ equal to 5% (Davis and Haltiwanger, 1990). Moreover, we assume that unemployment benefits $b = rep_b W$ as well as the steady state leisure gain from unemployment

$$\frac{\psi_f}{\lambda_{c,t}} \frac{h_t^{1+\sigma_f}}{1+\sigma_f} = rep_h W = rep_h \lambda_y (1-\alpha)(Y/N), \quad (3.42)$$

can be quantified as percentage of the competitive price of labor services W . This allows us to derive an expression which we can solve for the steady state (un)employment rate in closed form. Therefore, we plug the vacancy filling rate (3.31), the steady state job flow condition (3.3), and the wage equation (3.37) into the job creation condition (3.31):

$$\frac{N}{1-N} = \frac{[(1-\eta^*)(1-\alpha)(1-(rep_b + rep_h))\lambda_y Y] - [(\kappa V/\beta\rho)(1-\beta(1-\rho))]}{\eta^* \kappa V} \quad (3.43)$$

We parameterize equation (3.43) as follows: *Effective* bargaining power is assumed to be symmetrically distributed, i.e. $\eta^* = 0.5$ (Svejnar, 1986). Unemployment benefits $rep_b = 0.36$ are calibrated using OECD (2006, p. 60) data on the net replacement rate. Average vacancy posting costs are set to the standard value of $\kappa V = 1\%$ (Hamermesh and Pfann, 1996, p. 1278). Output Y is normalized to unity. Given these values, we have to set $rep_h = 28.5\%$ in order to replicate an average unemployment rate $U = 5.7\%$. Thus, the total replacement ratio is equal to $rep_b + rep_h = 64.5\%$. Our calibration implies that the semi-elasticity of unemployment with respect to the replacement rate is equal to 3. This value lies within the range of estimates by Costain and Reiter (2008).¹⁰

The matching elasticity of vacancies ($\mu = 0.5$) does not affect the steady state of the model economy, but its cyclical behavior. We set $\mu = 0.5$, which is within the interval $[0.3, 0.5]$ proposed by Petrongolo and Pissarides (2001). Moreover, our choice ($\mu = \eta^*$) satisfies the Hosios (1990) condition. Finally, we set the steady state vacancy filling rate $q(\theta)$ to 0.7 (van Ours and Ridder, 1992).¹¹

Right-to-Manage Bargaining Given that average working time h is normalized to unity (Trigari, 2006), we can derive an expression for labor efficiency σ_h , using the competitive price of labor services (3.25), the value of a job to the firm (3.29), the firm's first

¹⁰As a further robustness check, we reduce unemployment benefits by 10 percentage points. This implies that the steady state unemployment rate falls by 1.3 percentage points, which is in line with the results of Bassanini and Duval (2006).

¹¹As demonstrated by Shimer (2005), the model allows for the normalization of the vacancy filling rate. Nevertheless, we choose a meaningful value.

order condition for hours per worker (3.32), and the job creation condition (3.31):

$$\sigma_h = 0.9775 = 1 - \frac{(\kappa V / \rho \beta)(1 - \beta(1 - \rho))}{\lambda_y(1 - \alpha)H(h)}. \quad (3.44)$$

This value is close to constant returns to scale (Christoffel and Kuester, 2008). We then assume that worker's "nominal" and effective bargaining power are equal in the steady state, i.e. $\eta = \eta^* = 0.5$. This implies that, in the steady state, the net marginal benefit of an increase in the real wage rate to the worker (δ^W) equals the net marginal loss to the hiring firm (δ^F). According to equations (3.35) and (3.36), this condition holds if the real wage rate w equals the marginal rate of substitution. This requires setting $\sigma_h = 2.43$, a value that is consistent with the results of MaCurdy (1983).

Government and Monetary Policy We calibrate the share of governmental consumption in total output g to 18% (Smets and Wouters, 2007), which implies an average consumption share (C/Y) of about 56%. The ratio of nominal output $P Y$ to the monetary aggregate M , i.e. the velocity of money, is set to 0.36 (Christiano et al., 2005). The values chosen for the generalized Taylor rule ($\phi_r = 0.8, \phi_\pi = 2.0, \phi_y = 0.3$) are taken from Gertler et al. (2008).

Price and Wage Rigidities We adopt the Calvo (1983) price ($\omega_p = 0.60$) and wage ($\omega_w = 0.65$) rigidity parameters estimated by Christiano et al. (2005).

Stochastic Processes We calibrate the law of motion for the technology shock using the conventional values ($\rho^z = 0.95, \iota^z = 0.007$) suggested by Cooley and Prescott (1995). The standard deviation of the monetary policy shock ($\iota^m = 0.002$) is taken from Walsh (2005).

3.3.2 Calibration France

Our analysis focuses on the impact of staggered prices, staggered wages, and the size of labor market flows. Hence, in order to facilitate comparability with the US model economy, we only alter the respective parameters (Table 3.2). Following Álvarez et al. (2006), we set the degree of price rigidity $\omega_p^F = 0.75$. The parameter governing the degree of wage rigidity ($\omega_w^F = 0.83$) is chosen in accordance with du Caju et al. (2008). Furthermore, we target the average French unemployment rate between 1978:2007, i.e. $U^F = 9.0\%$ (OECD, 2008b). Given that the French job finding rate exhibits almost no duration dependence (Hobijn and Sahin, 2007; Elsby et al., 2009), we approximate the steady-state job finding rate ($q^F(\theta^F)\theta^F = 21.3\%$) by the average fraction of workers unemployed for less than three months (OECD, 2008a). These values imply an average job

separation rate equal to $\rho^F = 2.36\%$.¹² The amount of French unemployment benefits is calibrated to $rep_b^F = 0.57$ (OECD, 2006). We then set the leisure gain from unemployment $rep_l^F = 0.188$ in order to match the average French unemployment rate (Equation 3.43). Finally, we choose $\sigma_f^F = 4.19$, such that $\eta^* = \eta$ holds. The implied value of labor efficiency ($\sigma_h^F = 0.9723$, Equation 3.44) remains almost unchanged.

3.3.3 Inspecting the Mechanism of Staggered Wage Contracts

This section examines the dynamic behavior of a New Keynesian business cycle model with right-to-manage wage bargaining. Our computations are performed using Dynare 4.0.2 (Juillard, 1996). Table (3.1) presents the impulse responses of the US model economy to an innovation in monetary policy, given different values of the real wage rigidity parameter ($\omega_w = \{0.00, 0.01, 0.65, 0.83\}$). Table (3.2) repeats the same exercise for a neutral technology shock. The graphs depict the evolution of the impulse responses over 32 quarters.

Impulse Responses to an Innovation in Monetary Policy The impulse responses reveal that staggered wage contracts are an effective means to reduce the elasticity of the *average* real wage rate. Even if there is only a very small share of matches that are unable to re-negotiate ($\omega_w = 0.01$), we observe a significant difference in the dynamic behavior of the real wage rate compared to the fully flexible wage regime ($\omega_w = 0.00$). If we increase the wage rigidity parameter until it equals the value estimated for the US economy ($\omega_w = 0.60$), the elasticity of the real wage rate decreases further. However, the value estimated for France ($\omega_w = 0.83$) generates almost the same impulse response as the US value.

Since labor efficiency σ_h is close to unity, the impulse responses of the average real wage rate w_t and of the competitive price of labor services W_t match each other almost exactly. Moreover, Equation (3.26) shows that the competitive price of labor services feeds directly into the determination of real marginal costs $\lambda_{y,t}$. Hence, any rigidity in the average real wage rate is transmitted via the competitive price of labor services into the dynamic time path of real marginal costs. This implies that real wage rigidity under right-to-manage wage bargaining is able to reduce the elasticity of real marginal costs. The New Keynesian Phillips Curve entails that these sluggish dynamics translate into persistent movements of inflation. The direct link between real wage rigidity and inflation persistence established by the right-to-manage bargaining model is known as the “wage channel” (Trigari, 2006).

Furthermore, we note that the model is not only capable to generate persistent responses in inflation, but also in output and total labor input. The so-called wage channel established by the right-to-manage bargaining approach increases not only the persistence

¹²Our values are almost identical to the ones of Sigrist (2009), who estimates an average job finding rate equal to 20.1% and an average job separation rate equal to 2.4% for France on a quarterly basis.

of inflation, but dampens also the response of the real interest rate. In the case of variable capital utilization, this implies that the input of capital services responds more sluggishly. Consequently, given that matching frictions induce a lagged response in total labor input, we note that the response of aggregate output reaches its minimum not impact, but just in the second period after an innovation in monetary policy.

The mechanism behind staggered right-to-manage wage bargaining can be described as follows. Firm j is able to set unilaterally the profit maximizing number of hour per worker $h_{j,t}$, given the spread between the real wage $w_{j,t}$ paid in match j and the competitive price of labor services W_t (Equation 3.33). Recall that the impulse responses of the average real wage rate w_t and of the competitive price of labor services W_t match each other almost exactly. Hiring firms that are unable to re-negotiate, however, may be forced to pay a real wage rate $w_{j,t}$ that is quite different from the competitive price of labor services. Hence, given that labor efficiency σ_h is close to unity, these firms tend to adjust the number of hours per worker drastically.¹³

Following three impulse response functions illustrate the consequences of right-to-manage wage bargaining: (i) the number of hours per worker associated with the *average* real wage rate w_t ,¹⁴ (ii) the number of hours per worker associated with the *re-negotiated* real wage rate w_t^* , and (iii) the number of hours per worker associated with the *lagged* average real wage rate w_{t-1} . In the case of fully flexible real wages, the average real wage rate and the re-negotiated real wage rate are identical, and so are the corresponding impulse responses of hours per worker. Yet, even if there is only a very small share of firms that is unable to re-negotiate ($\omega_w = 0.01$), we note that these firms find it optimal to adjust the number of hours per worker to a large extent. In response to an innovation in monetary policy, the impulse response of hours per worker associated with the lagged average real wage rate $h(w_{t-1})$ plummets on impact by more than 60% and then shoots up sharply, peaking at approximately 40% above its steady-state value after 3 quarters. Given that the average number of hours per worker $h(w_t)$ remains almost unchanged, large movements in $h(w_{t-1})$ imply a significant change in the adjustment pattern of hours per worker associated with the re-negotiated real wage rate $h(w_t^*)$. In the case of fully flexible wages, the impulse response of $h(w_t^*)$ shows a negative spike on impact and a fast convergence to its steady-state value. But if only 1% of the wage contracts is not re-negotiated in every period, the impulse response shows a clear hump-shape and a considerably slower speed of convergence.

We emphasize this issue, since the impulse response of the number of hours per worker associated with the re-negotiated real wage rate $h(w_t^*)$ is of great importance for the

¹³This is a distinct feature of the right-to-manage bargaining model with staggered wage contracts. Only if the number of hours per worker depends on the real wage rate, a dispersion of hours per worker can emerge.

¹⁴In our first order approximation, the number of hours per worker associated with the average real wage rate is equal to the average number of hours per worker (Schmitt-Grohe and Uribe, 2004).

dynamics of the whole model economy. In particular, the movements in $h(w_t^*)$ determine the sign of the response of the effective bargaining weight η_t^* . This implies that movements in $h(w_t^*)$ feed back into the re-negotiated real wage rate w_t^* . If wages are fully flexible, an innovation in monetary policy induces a fall in the effective bargaining weight of the household which accounts for approximately 2/3rd of the reduction in the re-negotiated real wage rate w_t^* . With increasing real wage rigidity, instead, the bargaining weight of the household *raises* on impact by 3% and, hence, stabilizes the re-negotiated real wage rate. This explains why the impulse response of the re-negotiated real wage w_t^* rate matches the impulse response of the average real wage rate w_t almost exactly.

Impulse Responses to a Neutral Technology Shock This section analyzes the effects of a neutral technology shock on the dynamic behavior of the model economy. Consistent with the results of Ravn and Simonelli (2008), Figure (3.2) shows that the impact response of employment is positive, and then continues to rise until it reaches a maximum after about 6 quarters.¹⁵ Real wage rigidity clearly amplifies the response of the employment level. Hours per worker, on the other hand, show a negative impact response. In the following periods, however, the average number of hours per worker rises dramatically and peaks after about 6 quarters. Comparing the elasticities of the employment level and hours per worker, we note that firms adjust employment primarily through the intensive margin. This prediction is not consistent with the data (see below). For this reason, we observe that the responses of total labor input and hours per worker are very similar. Total labor input remains below its steady-state value in the first few quarters after a neutral technology shock. As soon as prices adjust, intermediate good firms expand output and tend to demand more labor services from the hiring firms. Hence, total labor input follows a hump-shaped pattern. This implies that output and total labor input are positively correlated at the business cycle frequencies in response to a neutral technology shock ($\rho_{X,Y}=0.95$, see Table 3.3). Furthermore, the expansion in aggregate output induces the monetary authority to raise the nominal interest rate. Thus, we observe a pronounced U-shape in the impulse response of inflation.

Staggered wage contracts reduce the elasticity of the real wage in the same way as in response to monetary policy shocks. But, in contrast to the last section, real wage rigidity now increases the amplitude of the fluctuations in the average number of hours per worker. As a result, the elasticity of average labor costs $w_t h_t$ rises, the more rigid is the real wage rate. This surprising outcome is due to the fact that hiring firms that are unable to re-negotiate tend to increase the number of hours per worker $h_{j,t}$ enormously. Hiring firms that are able to re-negotiate, instead, even decrease the number of hours per worker h_t^* slightly.

¹⁵In our model, $N_t = 1 - U_t$ holds (see Footnote 3). Hence, the responses of employment and unemployment are symmetric.

In order to study the consequences of right-to-manage wage bargaining for the dynamics of the labor market, we recall that the incentive of a potential hiring firm to open a new vacancy is provided by the discounted flow of expected profits. Moreover, as shown by Christoffel and Kuester (2008), right-to-manage bargaining entails that the profit flow of a hiring firm $\Pi_{j,n}$ is proportional to its labor costs $w_{j,t}h_{j,t}$:

$$\Pi_{j,n}(t) = W_t H(h_{j,t}) - w_{j,t}h_{j,t} = ((1 - \sigma_h)/\sigma_h)w_{j,t}h_{j,t} \quad (3.45)$$

In other words, the model predicts that (un)employment fluctuates stronger, the more volatile are labor costs. For this reason, the introduction of real wage rigidity amplifies the volatility of the labor market variables. It does so, however, not because labor costs are more rigid, but because labor costs are more volatile.

In addition to that, we note that real wage rigidity raises the elasticity of output by a large extent. The increase in the elasticity of aggregate output is mainly driven by adjustments in the average number of hours per worker. Since firms are able to adjust the number of hours per worker unilaterally, they do so extensively. This explains why right-to-manage bargaining model with staggered wage contracts is able to increase the *absolute* volatility of the labor market, but not the *relative* movements of unemployment with respect to aggregate output (see also Table 3.3). Hence, our model cannot replicate the stylized business cycle fact that most of the variation in total labor input is due to movements into and out of employment rather than to adjustments in the average number of hours per worker (see Section 3.3.4).

This finding clearly contradicts previous work by Shimer (2004) and Hall (2005), which suggests that real wage rigidity establishes an important amplification mechanism for the labor market. The opposing implications are driven by differences in the underlying bargaining process. Under Nash bargaining, as assumed by Shimer (2004) and Hall (2005), the real wage rate w_t splits the mutual surplus, while hours per worker h_t are set independently of the actual real wage rate in order to maximize the mutual surplus. Maximization of the mutual surplus requires that the marginal product of labor is equal to the marginal rate of substitution (Cheron and Langot, 2004). This has three important implications. First, as shown by Trigari (2006), the marginal rate of substitution is the main determinant of the dynamics of real marginal costs — and not the real wage rate. Second, the profit flow of hiring firms is not proportional to labor costs. Consequently, models with Nash bargaining and real wage rigidity (Krause and Lubik, 2007) do not exhibit a “wage channel”, but are capable to amplify the relative volatility of the labor market. Third, the real wage rate is *not* allocative for hours per worker. Hence, the effective bargaining weight is constant and unable to absorb any shocks.

3.3.4 A Transatlantic Perspective

This section examines the impact of country-specific frictions on the dynamic behavior of the model economy. In particular, we focus on differences in the price rigidity parameter, the wage rigidity parameter, and in the degree of matching frictions in the labor market. In particular, we account for the fact that European transition rates between employment and unemployment are considerably lower. The higher value of the average unemployment rate is mainly due to a more generous replacement ratio. We then evaluate the model calibrated to the US (Section 3.3.1) against the model calibrated to the French economy (Section 3.3.2). In order to disentangle the effects of these frictions, we additionally evaluate two counter-factual model economies: (i) the model calibrated to France, but with prices flexible as in the US and (ii) the model calibrated to France, but with prices *and* wages flexible as in the US. The latter model exhibits the same Calvo type rigidity parameters as the US model economy, but differs in the calibration of the labor market.

Impulse Responses to an Innovation in Monetary Policy Table (3.3) shows that the degree of price rigidity plays a dominant role in the determination of aggregate inflation. If prices change more frequently, the impulse response function is considerably more elastic and immediate. The more flexible response of US prices entails that aggregate output falls by less and converges much faster to its steady state value. Quite surprisingly, the higher degree of real wage rigidity in France has no significant impact on the responses of inflation and output. Furthermore, we observe that the impulse response of the French unemployment rate exhibits a clear hump-shape. In the US model economy, on the contrary, the unemployment rate spikes on impact and then converges quickly to its steady state value. This pattern is mainly explained by the smaller value of the job separation rate which delays labor market turnover.

In summary, the model indicates that the transmission of an innovation in monetary policy to the economy is mainly determined by the degree of price rigidity. The degree of real wage rigidity, in contrast, seems to be less important.¹⁶ In addition, we find out that central banks concerned about the stabilization of employment should closely monitor the transition rates between the different labor market states.

Impulse Responses to a Neutral Technology Shock When the two model economies are hit by a neutral technology shock (Table 3.4), we observe more interdependencies between the three frictions considered. On the one hand, the larger degree of price rigidity raises the amplitude of output and inflation. On the other hand, the larger degree of real wage rigidity dampens the amplitude and delays the speed of convergence. In total, the

¹⁶Section (3.3.3) sheds some light upon this surprising result. The presence of real wage rigidity is relevant for the transmission of monetary policy. However, the medium US value and the high French value generate almost the same results

amplitude of both impulse responses remains almost constant, but convergence is slower under the French calibration. Again, the labor market calibration affects primarily the response of the unemployment rate. First, we note that the percentage impact response of the French unemployment rate is only about $1/4$ of the US value. Second, in the same way as above, greater price rigidity increases the amplitude of the unemployment rate, while a large degree of real wage rigidity dampens the fluctuations. As a result, the joint impact of the two Calvo type rigidities raises the persistence of the unemployment rate, but leave its amplitude virtually unchanged.

Discussion of the Second Moments Table (3.3) illustrates the unconditional second moments of the US economy, the French economy, and the conditional model generated data. As is well known, US labor market fluctuations are very volatile and persistent. The US unemployment rate is about 7 times as volatile as output, vacancies even more. This stylized fact has attracted much attention in the recent literature.¹⁷ Total labor input is about as volatile as output. Most of its variability seems to be due to variations in the stock of employment rather than the average number of hours per worker, confirming the findings of Cooley and Prescott (1995). The wage bill per worker is significantly less volatile than output. Besides, we observe that consumption is somewhat less volatile than output, while investment fluctuates more. Inflation exhibits significantly less cyclical variability than output, is counter-cyclical, and very persistent.

Quite surprisingly, we notice that the unconditional moments of the French economy are fairly similar. The most interesting differences are the following. The volatility of French output is only about $2/3$ of the US value. This implies that the absolute volatility of aggregate variables like unemployment, vacancies, investment or consumption is significantly lower than in the US, although the relative volatilities are very close to each other. In addition, we note that, in France, co-movement between output and all variables considered is weaker. In particular, the wage bill per worker and its components are essentially acyclical. Nevertheless, the wage bill per worker exhibits a considerable degree of cyclical volatility.

The model presented is not designed to match these facts. The model was rather developed to replicate the qualitative pattern of the impulse response functions. Nevertheless, the model is capable to replicate a positive correlation between output and total labor input at the business cycle frequencies to a neutral technology shock (Ravn and Simonelli, 2008). Apart from that, the simulated data clearly point out along which lines the fit of the model is yet to be improved. Neither the neutral technology shock, nor the monetary policy shock is able to explain the large cyclical volatility in the unemployment rate. On the other hand, the model generates excess volatility in the number of hours

¹⁷Shimer (2005) stimulated a considerable discussion on how to match the high volatility found in the data. The most prominent examples include staggered Nash bargaining (Gertler and Trigari, 2009) and an alternative calibration procedure (Hagedorn and Manovskii, 2008).

per worker. This implies that most of the volatility in total labor input is induced along the intensive margin. At least for the US, this is in contrast to the data. The paper by Christoffel and Kuester (2008) shows that the introduction of a per-period fixed costs in the production of labor services (representing, for instance, health insurance contributions) may help to increase the elasticity of the extensive margin. Another shortcoming of the staggered right-to-manage wage bargaining model is that even modest Calvo type rigidities in wage bargaining entail almost constant real wage rates over the business cycle.

3.4 Conclusion

This paper develops a New Keynesian business cycle model akin to Christiano et al. (2005) and Smets and Wouters (2003) with staggered right-to-manage wage bargaining (Trigari, 2006). We assume that, upon matching, firm-worker pairs first bargain over the real wage rate which is subject to staggered wage contracts. In the second step, hiring firms may choose the number of hours per worker unilaterally. This setting implies that the real wage rate is allocative for the number of hours per worker. Consequently, any rigidity in the real wage rate is transmitted via the New Keynesian Phillips Curve into persistent movements of inflation. This feature of the right-to-manage wage bargaining is referred to as the “wage channel”.

The key result of our paper is that a reasonably calibrated version of the model is able to generate persistent responses in output, inflation, and total labor input to both technology and monetary policy shocks. New Keynesian models with Nash bargaining (e.g. Walsh, 2005), in contrast, are not able to generate hump-shaped responses to monetary policy shocks once capital accumulation is introduced (Heer and Maussner, 2007). Staggered right-to-manage wage bargaining, however, increases not only the persistence of inflation, but also of the real interest rate. Since we assume variable capital utilization, this leads to a hump-shaped decline in the input of capital services. In addition to that, matching frictions induce a sluggish response in total labor input. Consequently, we observe that the response of aggregate output reaches its minimum not impact, but just in the second period after an innovation in monetary policy.

In response to a neutral technology shock, our model replicates a hump-shaped response of output and a U-shaped response of inflation. Turning to the labor market, we note that unemployment shows a negative impact response and then continues to decrease sluggishly. Hours per worker, instead, exhibit a negative impact response, but then rise for about 2 years before eventually falling. Hence, consistent with the findings of Ravn and Simonelli (2008), we observe a positive correlation between output and total labor input at the business cycle frequencies in response to a neutral technology shock.

Furthermore, we compare the model’s dynamic behavior when calibrated to the US and to a European economy. We find that the degree of price rigidity explains most

of the differences in response to a monetary policy shock. Differences in the degree of wage rigidity, instead, alter the dynamics of the model economy only by little. When the economy is hit by a neutral technology shock, both price and wage rigidities turn out to be important. Apart from that, our results indicate that matching frictions matter primarily for the dynamics of the labor market.

On the other hand, neither the neutral technology shock, nor the monetary policy shock is able to explain the large cyclical volatility in the unemployment rate. This implies that most of the volatility in total labor input is induced along the intensive margin. At least for the US, this is in contrast to the data. The paper by Christoffel and Kuester (2008) shows that the introduction of a per-period fixed costs in the production of labor services (representing, for instance, health insurance contributions) may help to increase the elasticity of the extensive margin. Another shortcoming of the staggered right-to-manage wage bargaining model is that even modest Calvo type rigidities in wage bargaining entail almost constant real wage rates over the business cycle.

It would be interesting to extend our analysis along two dimensions. First, we have only investigated the impact of two structural shocks so far. Therefore, it seems to be a natural choice to extend our analysis to a variety of other shocks. In particular, the literature suggests examining the impact of investment-specific technology shock, government spending shocks, or shock to the matching technology. The second step in our research program will be to estimate the present model along the lines described by Smets and Wouters (2007).

Appendix 3.A The Log-Linear Model

$$\hat{M}_t = -\mu(N/U)\hat{N}_{t-1} + (1-\mu)\hat{V}_t \quad \text{ad (3.1)}$$

$$\hat{q}_t = \hat{M}_t - \hat{V}_t \quad \text{ad (3.2)}$$

$$N\hat{N}_t = (1-\rho)N\hat{N}_{t-1} + M\hat{M}_t \quad \text{ad (3.3)}$$

$$\hat{w}_t = \omega_w \hat{w}_t^* + (1-\omega_w)\hat{w}_{t-1} \quad \text{ad (3.4)}$$

$$K\hat{K}_t = (1-\delta)K\hat{K}_{t-1} + I\hat{I}_t \quad \text{ad (3.8)}$$

$$\hat{\lambda}_{c,t} = [\sigma_c / ((1-\psi_c)(1-\beta\psi_c))] [\psi\hat{C}_{t-1} - (1+\beta\psi_c^2)\hat{C}_t + \psi_c\beta E_t\{\hat{C}_{t+1}\}] \quad \text{ad (3.12)}$$

$$r\hat{r}_t = \chi_a \hat{x}_t \quad \text{ad (3.13)}$$

$$\hat{\lambda}_{c,t} = \hat{R}_t + E_t\{\hat{\lambda}_{c,t+1} - \pi_{t+1}\} \quad \text{ad (3.14)}$$

$$\hat{I}_t = [(\hat{\lambda}_{k,t} - \hat{\lambda}_{c,t})/\chi_s] + \hat{I}_{t-1} + \beta E_t\{\hat{I}_{t+1}\} / (1+\beta) \quad \text{ad (3.16)}$$

$$\hat{Q}_t = \hat{\lambda}_{k,t} - \hat{\lambda}_{c,t} \quad \text{ad (3.17)}$$

$$\hat{Q}_t = E_t\{\hat{\lambda}_{c,t+1} - \hat{\lambda}_{c,t} + \beta(1-\delta)\hat{Q}_{t+1} + \beta r\hat{r}_{t+1}\} \quad \text{ad (3.17)}$$

$$\begin{aligned} \mathcal{W}\hat{W}_t^* &= \frac{wh}{1-\sigma_h} (\hat{W}_t - \sigma_h \hat{w}_t^*) - \frac{\psi_f h^{1+\sigma_f}}{(1-\sigma_h)\lambda_c} (\hat{W}_t - \hat{w}_t^*) + \frac{\psi_f h^{1+\sigma_f}}{(1+\sigma_f)\lambda_c} \hat{\lambda}_{c,t} \quad \text{ad (3.18)} \\ &+ \frac{\beta(1-\rho)\omega_w}{1-\beta(1-\rho)\omega_w} \left(\frac{\sigma_h wh}{1-\sigma_h} - \frac{\psi_f h^{1+\sigma_f}}{\lambda_c(1-\sigma_h)} \right) E_t\{\hat{w}_{t+1}^* - \hat{w}_t^*\} \\ &- \frac{\beta q(\theta)\theta\omega_w}{1-\beta(1-\rho)\omega_w} \left(\frac{\sigma_h wh}{1-\sigma_h} - \frac{\psi_f h^{1+\sigma_f}}{\lambda_c(1-\sigma_h)} \right) E_t\{\hat{w}_{t+1}^* - \hat{w}_t\} \\ &+ (1-\rho-q(\theta)\theta)\beta\mathcal{W}E_t\{\hat{\lambda}_{c,t+1} - \hat{\lambda}_{c,t} + \hat{\mathcal{W}}_{t+1}^*\} - \beta\mathcal{W}q(\theta)\theta(\hat{M}_t + (N/U)\hat{N}_{t-1}) \end{aligned}$$

$$\hat{Y}_t = \hat{\epsilon}_t^z + \alpha\hat{K}_{t-1} + \alpha\hat{x}_t + (1-\alpha)\hat{N}_{t-1} + (1-\alpha)\sigma_h\hat{h}_t \quad \text{ad (3.22)}$$

$$\hat{\epsilon}_t^z = \rho^z \hat{\epsilon}_{t-1}^z + \hat{l}_t^z \quad \text{ad (3.23)}$$

$$\hat{r}_t = \hat{\lambda}_{y,t} + \hat{Y}_t - \hat{K}_{t-1} - \hat{x}_t \quad \text{ad (3.24)}$$

$$\hat{W}_t = \hat{\lambda}_{y,t} + \hat{Y}_t - \hat{N}_{t-1} - \sigma_h\hat{h}_t \quad \text{ad (3.25)}$$

$$\hat{\pi}_t = (\hat{\pi}_{t-1} + \beta E_t\{\hat{\pi}_{t+1}\} + [((1-\beta\omega_p)(1-\omega_p))/\omega_p]\hat{\lambda}_{y,t}) / (1+\beta) \quad \text{ad (3.27)}$$

$$\mathcal{J}\hat{\mathcal{J}}_t^* = \frac{wh}{\sigma_h} (\hat{W}_t - \sigma_h \hat{w}_t^*) \quad \text{ad (3.29)}$$

$$+ \frac{wh\beta(1-\rho)\omega_w}{1-\beta(1-\rho)\omega_w} E_t\{\hat{w}_{t+1}^* - \hat{w}_t^*\} + \beta(1-\rho)\mathcal{J}E_t\{\hat{\lambda}_{c,t+1} - \hat{\lambda}_{c,t} + \hat{\mathcal{J}}_{t+1}^*\}$$

$$- \frac{\kappa}{q}\hat{q}_t = \left[\frac{wh\beta\omega_w}{1-\beta(1-\rho)\omega_w} \right] E_t\{\hat{w}_{t+1}^* - \hat{w}_t\} + \beta\mathcal{J}E_t\{\hat{\lambda}_{c,t+1} - \hat{\lambda}_{c,t} + \hat{\mathcal{J}}_{t+1}^*\} \quad \text{ad (3.31)}$$

$$\hat{w}_t = \hat{W}_t - (1-\sigma_h)\hat{h}_t \quad \text{ad (3.32)}$$

$$\hat{\mathcal{J}}_t^* = \hat{\mathcal{W}}_t^* + \hat{\delta}_t^F - \hat{\delta}_t^W \quad \text{ad (3.34)}$$

$$\delta^W \hat{\delta}_t^W = -\frac{wh\sigma_h}{(1-\sigma_h)^2} (\hat{W}_t - \sigma_h \hat{w}_t^*) + \frac{(1+\sigma_f)\psi_f h^{1+\sigma_f}}{(1-\sigma_h)^2 \lambda_c} (\hat{W}_t - \hat{w}_t^*) - \frac{\psi_f h^{1+\sigma_f}}{(1-\sigma_h)\lambda_c} \hat{\lambda}_{c,t}$$

ad (3.35)

$$+ \beta(1-\rho)\omega_w \delta^W E_t \left\{ \left(\frac{(1+\sigma_f)\psi_f h^{\sigma_f}}{(1-\sigma_h)^2 \lambda w} - \left(\frac{\sigma_h}{1-\sigma_h} \right)^2 \right) (\hat{w}_{t+1}^* - \hat{w}_t^*) \right\}$$

$$+ \beta(1-\rho)\omega_w \delta^W E_t \left\{ \hat{\delta}_{t+1}^W + \hat{\lambda}_{c,t+1} - \hat{\lambda}_{c,t} \right\}$$

$$\delta^F \hat{\delta}_t^F = \frac{wh}{1-\sigma_h} (\hat{W}_t - \sigma_h \hat{w}_t^*)$$

ad (3.36)

$$+ \beta(1-\rho)\omega_w \delta^F E_t \left\{ \frac{\sigma_h}{1-\sigma_h} (\hat{w}_{t+1}^* - \hat{w}_t^*) + \hat{\delta}_{t+1}^F + \hat{\lambda}_{c,t+1} - \hat{\lambda}_{c,t} \right\}$$

$$\hat{R}_t = \phi_r \hat{R}_{t-1} + (1-\phi_r)\phi_\pi \hat{\pi}_t + (1-\phi_r)\phi_y \hat{Y}_t + \hat{\epsilon}_t^r$$

ad (3.40)

$$\hat{\epsilon}_t^r = \rho^r \hat{\epsilon}_{t-1}^r + \hat{\iota}_t^r$$

ad (3.40)

$$Y\hat{Y}_t = I\hat{I}_t + C\hat{C}_t + \kappa V\hat{V}_t + G\hat{Y}_t + K\chi_b \hat{x}_t$$

ad (3.41)

Appendix 3.B Tables

Parameter	Description	Value	Source
Preferences			
β	discount factor	0.99	Kydland and Prescott (1982)
σ_c	relative risk aversion	1	Christiano et al. (2005)
ψ_c	habit formation	0.65	Christiano et al. (2005)
σ_q	money demand elasticity	6.3	Christiano et al. (2005)
ξ_p	elasticity of substitution	6	Christiano et al. (2005)
σ_f	hours supply elasticity	2.43	MaCurdy (1983)
Production and the Capital Market			
α	capital elasticity	0.36	Christiano et al. (2005)
χ_s	investment adjustment cost	2	Christiano et al. (2005)
χ_b/χ_a	utilization elasticity	1	Smets and Wouters (2007)
δ	depreciation rate	0.025	Kydland and Prescott (1982)
Matching and the Labor Market			
U	unemployment rate	0.057	Shimer (2005)
$q(\theta)\theta$	job finding rate	0.828	Shimer (2005)
ρ	job destruction rate	0.05	Davis and Haltiwanger (1990)
η^*	effective bargaining power	0.5	Svejnar (1986)
rep_b	unemployment benefits	0.36	OECD (2006)
κV	vacancy posting costs	0.01	Hamermesh and Pfann (1996)
rep_h	leisure gain from U	0.285	Costain and Reiter (2008)
μ	matching elasticity of U	0.5	Petrongolo and Pissarides (2001)
$q(\theta)$	vacancy filling rate	0.7	van Ours and Ridder (1992)
Right-to-Manage Bargaining			
σ_h	labor efficiency	0.9775	Christoffel and Kuester (2008)
h	hours per worker	1	Trigari (2006)
η	“nominal” bargaining power	0.5	Nash (1953)
Government and Monetary Policy			
g	governmental consumption	0.18	Smets and Wouters (2007)
$(PY)/M$	velocity of money	0.36	Christiano et al. (2005)
ϕ_r	autoregressive parameter	0.8	Gertler et al. (2008)
ϕ_π	Taylor principle parameter	2.0	Gertler et al. (2008)
ϕ_y	output gap parameter	0.3	Gertler et al. (2008)
Price and Wage Rigidity			
ω_p	price rigidity	0.60	Christiano et al. (2005)
ω_w	wage rigidity	0.65	Christiano et al. (2005)
Stochastic Processes			
ρ^z	technology shock persistence	0.95	Cooley and Prescott (1995)
ι^z	technology shock sd	0.007	Cooley and Prescott (1995)
ι^m	monetary policy shock sd	0.002	Gertler et al. (2008)

Table 3.1: The parameterized US model economy

Parameter	Description	Value	Source
ω_p^F	price rigidity	0.75	Álvarez et al. (2006)
ω_w^F	wage rigidity	0.83	du Caju et al. (2008)
U^F	unemployment rate	0.091	OECD (2008 <i>b</i>)
$q^F(\theta^F)\theta^F$	job finding rate	0.212	OECD (2008 <i>a</i>)
ρ^F	job destruction rate	0.021	OECD (2008 <i>a,b</i>)
rep_b^F	unemployment benefits	0.57	OECD (2006)
rep_h^F	leisure gain from U	0.188	implied
σ_f^F	hours supply elasticity	4.19	implied
σ_h^F	labor efficiency	0.9723	implied

Table 3.2: Parameters specific to the French model economy

		Y	π	U	N	$N \times l$	l	w	$w \times l$	V	I	C
US Data	σ_X/σ_Y	(0.015)	0.59	7.32	0.80	0.96	0.26	0.50	0.54	9.03	2.88	0.82
	$\rho_{X,Y}$	1.00	-0.64	-0.86	0.81	0.87	0.70	0.20	0.52	0.89	0.94	0.85
	$\rho_{X_t,X_{t+1}}$	0.86	0.93	0.89	0.93	0.93	0.93	0.82	0.84	0.90	0.90	0.86
US Technology Shock	σ_X/σ_Y	(0.03)	0.37	0.68	0.04	1.03	0.99	0.13	1.08	0.90	2.14	0.34
	$\rho_{X,Y}$	1.00	-0.64	-0.91	0.91	0.95	0.94	0.67	0.95	0.71	0.99	0.98
	$\rho_{X_t,X_{t+1}}$	0.94	0.86	0.92	0.92	0.87	0.87	0.97	0.93	0.89	0.95	0.93
US Monetary Policy Shock	σ_X/σ_Y	(0.003)	0.29	1.50	0.09	1.24	1.18	0.03	1.18	3.06	1.74	0.41
	$\rho_{X,Y}$	1.00	0.79	-0.89	0.89	1.00	1.00	0.75	1.00	0.54	0.99	0.99
	$\rho_{X_t,X_{t+1}}$	0.74	0.87	0.53	0.53	0.74	0.73	0.84	0.73	0.13	0.78	0.72
French Data	σ_X/σ_Y	(0.009)	0.98	6.19	0.58	0.72	0.56	0.84	0.65	8.18	3.01	0.90
	$\rho_{X,Y}$	1.00	-0.49	-0.70	0.77	0.67	0.06	-0.08	-0.05	0.31	0.87	0.71
	$\rho_{X_t,X_{t+1}}$	0.87	0.90	0.90	0.95	0.93	0.91	0.83	0.77	0.77	0.90	0.73
France Technology Shock	σ_X/σ_Y	(0.02)	0.33	0.38	0.04	1.23	1.20	0.07	1.24	1.06	2.04	0.36
	$\rho_{X,Y}$	1.00	-0.64	-0.99	0.99	0.92	0.92	0.72	0.93	0.67	0.99	0.95
	$\rho_{X_t,X_{t+1}}$	0.95	0.92	0.96	0.96	0.87	0.86	0.97	0.87	0.92	0.96	0.95
France Monetary Policy Shock	σ_X/σ_Y	(0.004)	0.12	0.52	0.00	1.24	1.21	0.00	1.21	2.76	1.79	0.38
	$\rho_{X,Y}$	1.00	0.80	-0.96	0.96	1.00	1.00	0.56	1.00	0.66	0.99	0.99
	$\rho_{X_t,X_{t+1}}$	0.79	0.91	0.83	0.83	0.79	0.79	0.88	0.79	0.43	0.83	0.76

Table 3.3: Simulated Second Moments. For each variable, we report the relative standard deviation with respect to output σ_X/σ_Y , the co-movement with output $\rho_{X,Y}$, and the first order autocorrelation $\rho_{X_t,X_{t+1}}$. The percentage standard deviation of output is given in brackets. All data (1970:1-2008:4) are taken from the OECD databases “Economic Outlook” and “Main Economic Indicators”. The time series of French vacancies starts only in 1989:1. All time series are logged and de-trended with a Hodrick and Prescott (1997) filter 1600.

Appendix 3.C Impulse Response Functions

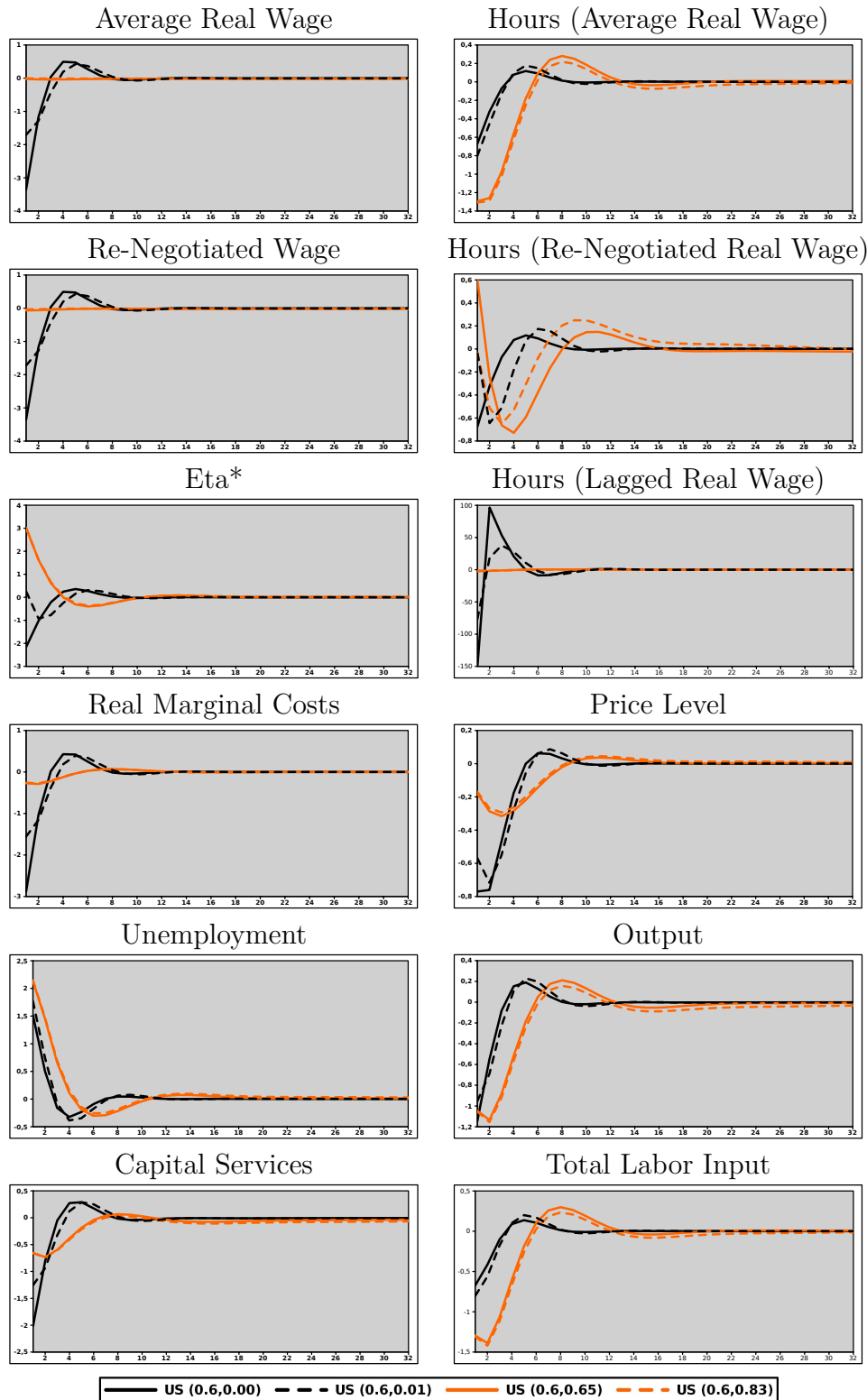


Figure 3.1: Impulse responses of the US model economy to a monetary policy shock. The black solid line represents the case $\omega_w = 0.00$. The back dashed line represents the case $\omega_w = 0.01$. The orange solid line represents the case $\omega_w = 0.65$. The orange dashed line represents the case $\omega_w = 0.83$. Units on the y -axis are given as percentage deviation from the steady state. Units on the x -axis correspond to quarters.

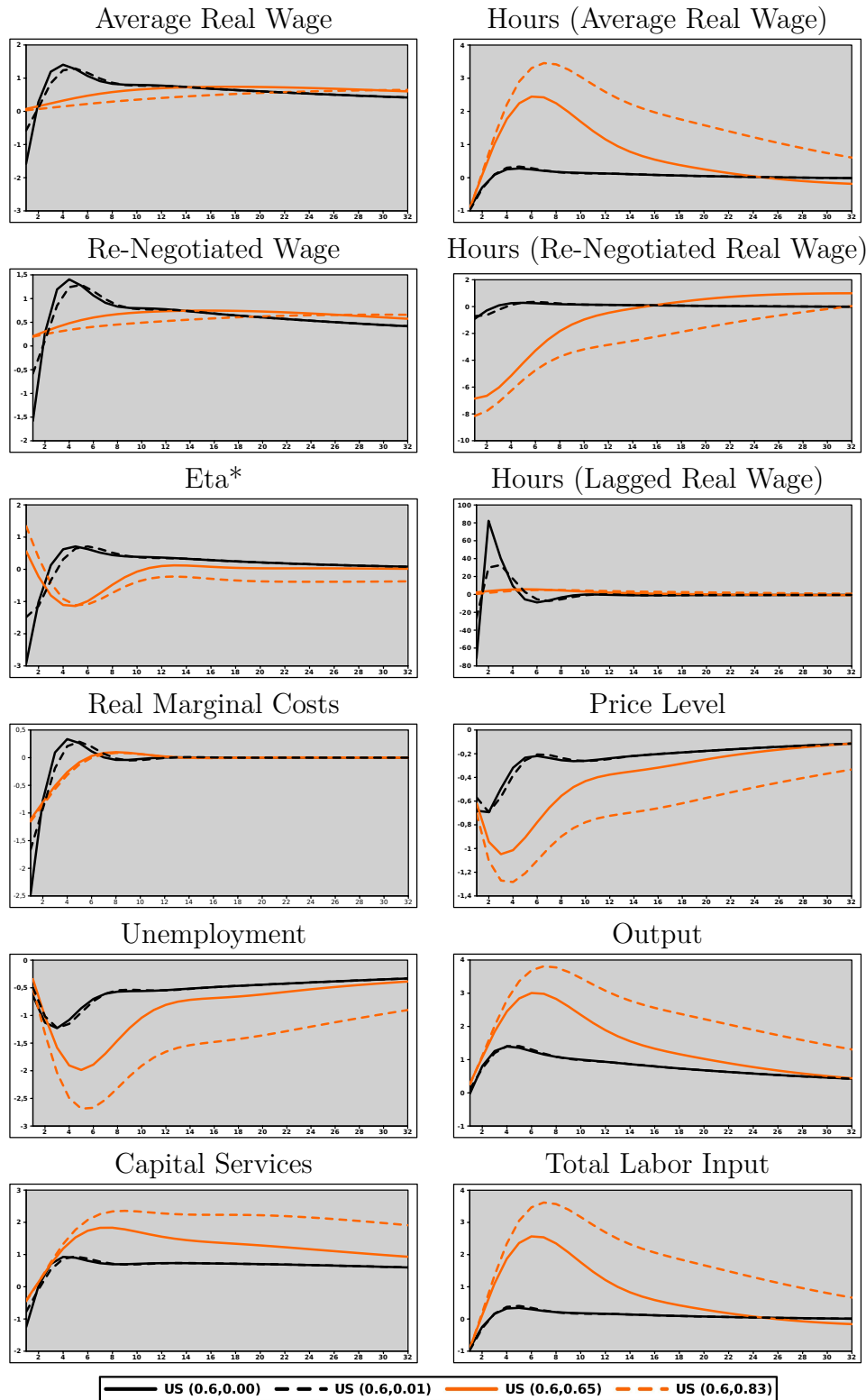


Figure 3.2: Impulse responses of the US model economy to a neutral technology shock. The black solid line represents the case $\omega_w = 0.00$. The back dashed line represents the case $\omega_w = 0.01$. The orange solid line represents the case $\omega_w = 0.65$. The orange dashed line represents the case $\omega_w = 0.83$. Units on the y -axis are given as percentage deviation from the steady state. Units on the x -axis correspond to quarters.

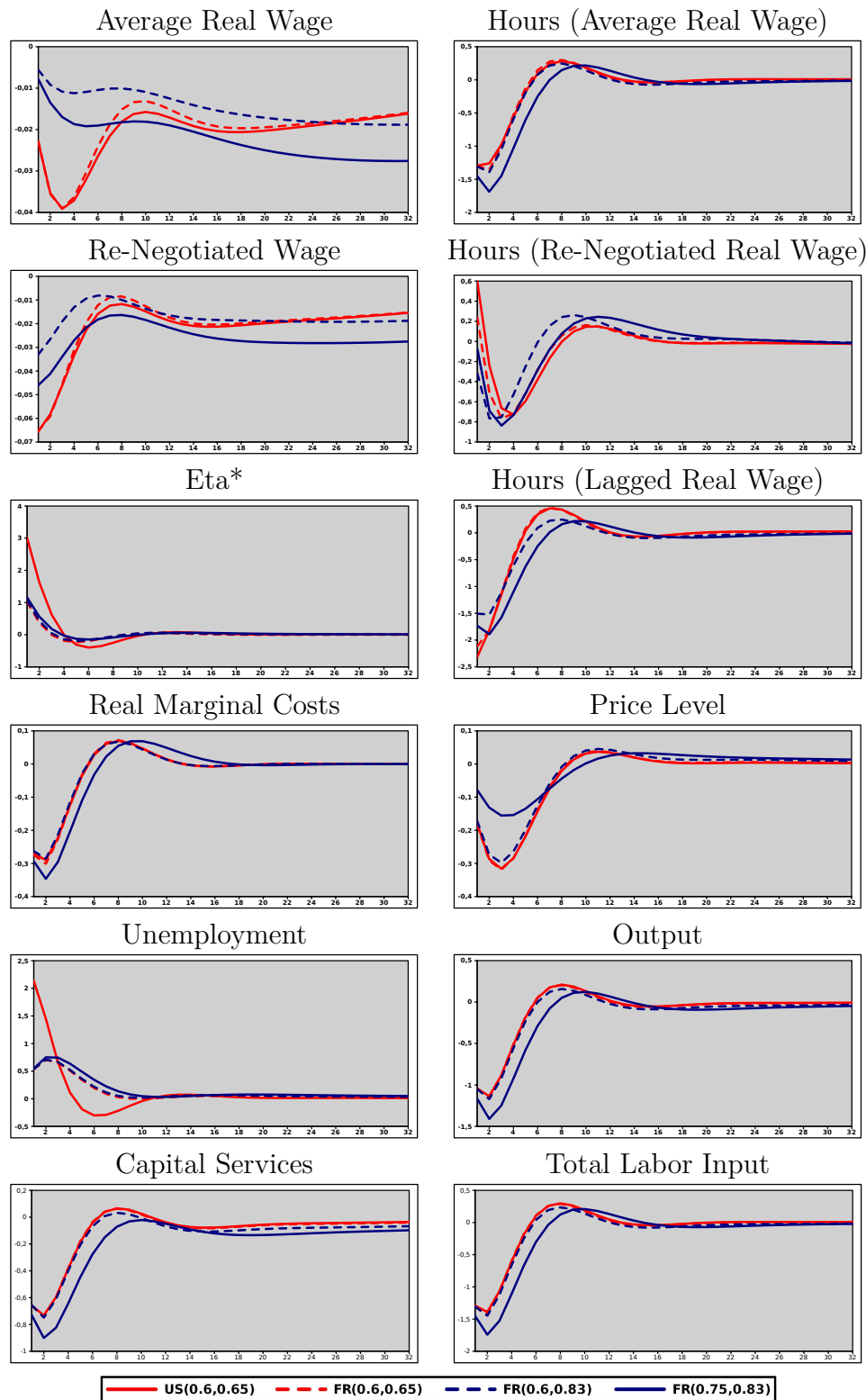


Figure 3.3: Impulse Responses to a Monetary Policy Shock. The red solid line represents the US model economy. The red dashed line represents the French model economy, but prices *and* wages are as flexible as in the US. The blue dashed line represents the French model economy with prices as flexible as in the US. The blue solid line represents the French model economy. Units on the y -axis are given as percentage deviation from the steady state. Units on the x -axis correspond to quarters.

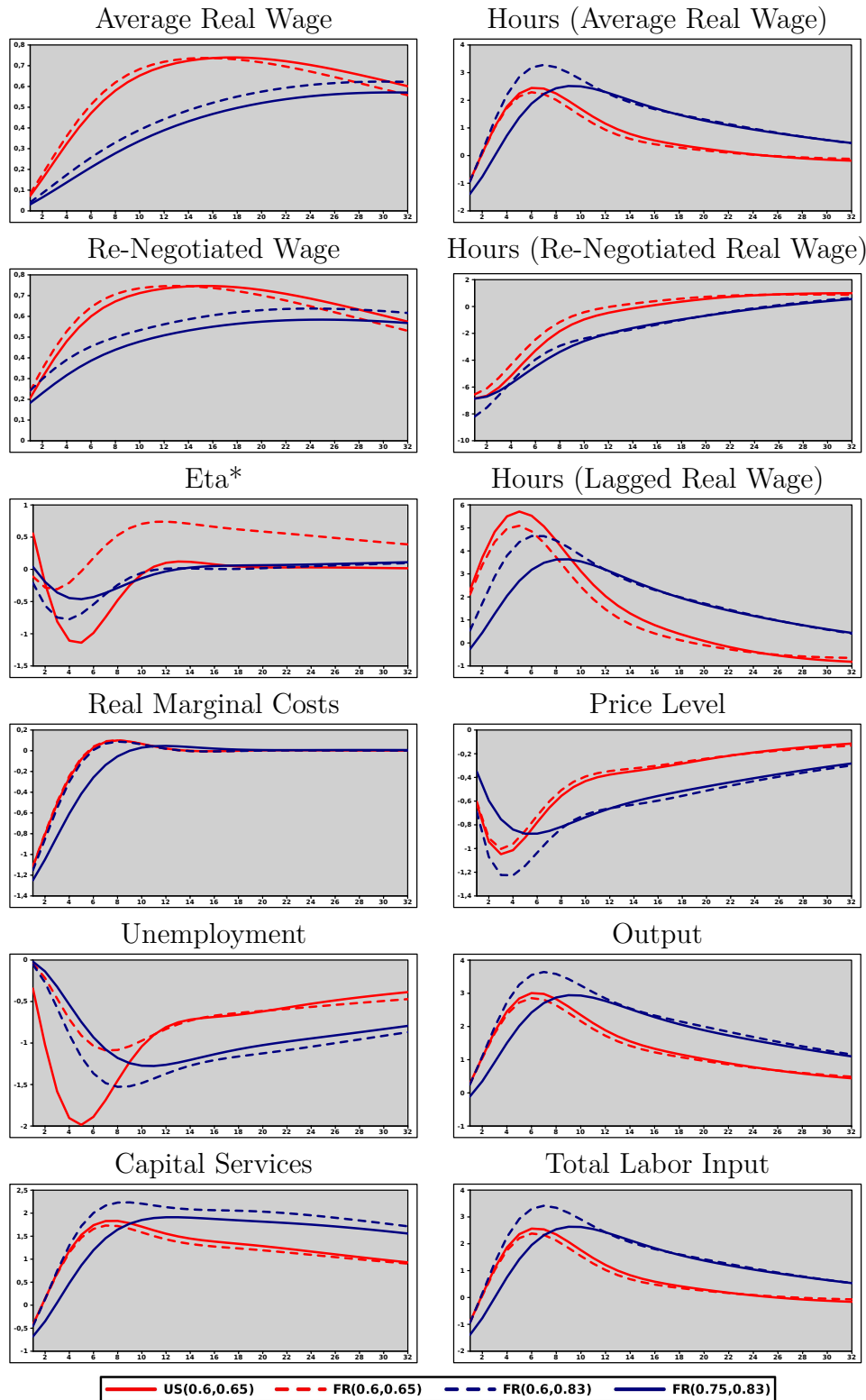


Figure 3.4: Impulse Responses to a Neutral Technology Shock. The red solid line represents the US model economy. The red dashed line represents the French model economy, but prices *and* wages are as flexible as in the US. The blue dashed line represents the French model economy with prices as flexible as in the US. The blue solid line represents the French model economy. Units on the y -axis are given as percentage deviation from the steady state. Units on the x -axis correspond to quarters.

Bibliography

- Álvarez, L. J., Dhyne, E., Hoeberichts, M., Kwapil, C., Bihan, H. L., Lünnemann, P., Martins, F., Sabbatini, R., Stahl, H., Vermeulen, P. and Vilmunen, J. (2006), ‘Sticky prices in the euro area: A summary of new micro-evidence’, *Journal of the European Economic Association* **4**(2-3), 575–584.
- Andolfatto, D. (1996), ‘Business cycles and labor-market search’, *American Economic Review* **86**(1), 112–132.
- Bassanini, A. and Duval, R. (2006), Employment patterns in OECD countries: Reassessing the role of policies and institutions, Working Papers No. 486, OECD Economics Department.
- Braun, H., Bock, R. D. and DiCecio, R. (2006), Aggregate shocks and labor market fluctuations, Working Papers No. 2006-004, Federal Reserve Bank of St. Louis.
- Calvo, G. A. (1983), ‘Staggered prices in a utility-maximizing framework’, *Journal of Monetary Economics* **12**(3), 383–398.
- Cheron, A. and Langot, F. (2004), ‘Labor market search and real business cycles: Reconciling Nash bargaining with the real wage dynamics’, *Review of Economic Dynamics* **7**(2), 476–493.
- Christiano, L. J., Eichenbaum, M. and Evans, C. L. (1999), Monetary policy shocks: What have we learned and to what end?, in J. B. Taylor and M. Woodford, eds, ‘Handbook of Macroeconomics’, Vol. 1 of *Handbook of Macroeconomics*, Elsevier, chapter 2, pp. 65–148.
- Christiano, L. J., Eichenbaum, M. and Evans, C. L. (2005), ‘Nominal rigidities and the dynamic effects of a shock to monetary policy’, *Journal of Political Economy* **113**(1), 1–45.
- Christiano, L. J., Eichenbaum, M. and Vigfusson, R. (2003), What happens after a technology shock?, NBER Working Papers No. 9819, National Bureau of Economic Research.
- Christiano, L. J., Eichenbaum, M. and Vigfusson, R. (2004), ‘The response of hours to a technology shock: Evidence based on direct measures of technology’, *Journal of the European Economic Association* **2**(2-3), 381–395.
- Christoffel, K., Costain, J., de Walque, G., Kuester, K., Linzert, T., Millard, S. and Pierard, O. (2009), Inflation dynamics with labour market matching: Assessing alternative specifications, Research series No. 200905-19, National Bank of Belgium.
- Christoffel, K. and Kuester, K. (2008), ‘Resuscitating the wage channel in models with unemployment fluctuations’, *Journal of Monetary Economics* **55**(5), 865–887.
- Clarida, R., Galí, J. and Gertler, M. (2000), ‘Monetary policy rules and macroeconomic stability: Evidence and some theory’, *The Quarterly Journal of Economics* **115**(1), 147–180.

- Cooley, T. F. and Prescott, E. C. (1995), Economic growth and business cycles, in T. F. Cooley, ed., 'Frontiers of Business Cycle Research', Princeton University Press, Princeton, New Jersey, pp. 1–38.
- Costain, J. S. and Reiter, M. (2008), 'Business cycles, unemployment insurance, and the calibration of matching models', *Journal of Economic Dynamics and Control* **32**(4), 1120–1155.
- Davis, S. and Haltiwanger, J. (1990), 'Gross job creation and destruction: Microeconomic evidence and macroeconomic implications', *NBER Macroeconomics Annual* **5**, 123–168.
- de Walque, G., Pierrard, O., Sneessens, H. and Wouters, R. (2008), Sequential bargaining in a New-Keynesian model with frictional unemployment and staggered wage negotiation, Working Papers No. 33, Central Bank of Luxembourg.
- du Caju, P., Gautier, E., Momferatou, D. and Ward-Warmedinger, M. (2008), Institutional features of wage bargaining in 22 EU countries, the US and Japan, Unpublished manuscript, Eurosystem Wage Dynamics Network.
- Elsby, M., Hobijn, B. and Sahin, A. (2009), Unemployment dynamics in the OECD, Working Paper Series 2009-04, Federal Reserve Bank of San Francisco.
- Francis, N. and Ramey, V. A. (2005), 'Is the technology-driven real business cycle hypothesis dead? Shocks and aggregate fluctuations revisited', *Journal of Monetary Economics* **52**(8), 1379–1399.
- Galí, J. (1999), 'Technology, employment, and the business cycle: Do technology shock-explain aggregate fluctuations?', *American Economic Review* **89**(1), 249–271.
- Gertler, M., Sala, L. and Trigari, A. (2008), 'An estimated monetary DSGE model with unemployment and staggered nominal wage bargaining', *Journal of Money, Credit and Banking* **40**(8), 1713–1764.
- Gertler, M. and Trigari, A. (2009), 'Unemployment fluctuations with staggered Nash wage bargaining', *Journal of Political Economy* **117**(1), 38–86.
- Haefke, C., Sonntag, M. and van Rens, T. (2009), Wage rigidity and job creation, Kiel Working Papers No. 1504, Kiel Institute for the World Economy.
- Hagedorn, M. and Manovskii, I. (2008), 'The cyclical behavior of equilibrium unemployment and vacancies revisited', *American Economic Review* **98**(4), 1692–1706.
- Hall, R. (2005), 'Employment fluctuations with equilibrium wage stickiness', *American Economic Review* **95**(1), 50–65.
- Hamermesh, D. S. and Pfann, G. A. (1996), 'Adjustment costs in factor demand', *Journal of Economic Literature* **34**(3), 1264–1292.
- Heer, B. and Maussner, A. (2007), Inflation and output dynamics in a model with labor market search and capital accumulation, Working Paper Series No. 2036, CESifo.
- Hobijn, B. and Sahin, A. (2007), Job-finding and separation rates in the OECD, Staff Report No. 298, Federal Reserve Bank of New York.

- Hodrick, R. J. and Prescott, E. C. (1997), ‘Postwar U.S. business cycles: An empirical investigation’, *Journal of Money, Credit and Banking* **29**(1), 1–16.
- Hosios, A. J. (1990), ‘On the efficiency of matching and related models of search and unemployment’, *Review of Economic Studies* **57**(2), 279–298.
- Juillard, M. (1996), Dynare: A program for the resolution and simulation of dynamic models with forward variables through the use of a relaxation algorithm, Working Papers (Couverture Orange) No. 9602, CEPREMAP.
- Krause, M. U. and Lubik, T. A. (2007), ‘The (ir)relevance of real wage rigidity in the New Keynesian model with search frictions’, *Journal of Monetary Economics* **54**(3), 706–727.
- Kydland, F. E. and Prescott, E. C. (1982), ‘Time to build and aggregate fluctuations’, *Econometrica* **50**(6), 1345–1370.
- Leeper, E. M., Sims, C. A. and Zha, T. (1996), ‘What does monetary policy do?’, *Brookings Papers on Economic Activity* **27**(1996-2), 1–78.
- MaCurdy, T. E. (1983), ‘A simple scheme for estimating an intertemporal model of labor supply and consumption in the presence of taxes and uncertainty’, *International Economic Review* **24**(2), 265–289.
- Merz, M. (1995), ‘Search in the labor market and the real business cycle’, *Journal of Monetary Economics* **36**(2), 269–300.
- Nash, J. (1953), ‘Two-person cooperative games’, *Econometrica* **21**, 128–140.
- OECD (2006), *Employment Outlook, Boosting Jobs and Incomes*, Organisation for Economic Co-Operation and Development.
- OECD (2008a), *Employment and Labour Market Statistics: Incidence of unemployment by duration*, Organisation for Economic Co-Operation and Development.
- OECD (2008b), *Employment and Labour Market Statistics: Labour force status by sex and age*, Organisation for Economic Co-Operation and Development.
- Peersman, G. and Straub, R. (2005), Technology shocks and robust sign restrictions in a euro area SVAR, Working Papers No. 05/288, Ghent University, Faculty of Economics and Business Administration.
- Petrongolo, B. and Pissarides, C. A. (2001), ‘Looking into the black box: A survey of the matching function’, *Journal of Economic Literature* **39**(2), 390–431.
- Pissarides, C. A. (1985), ‘Short-run equilibrium dynamics of unemployment, vacancies and real wages’, *American Economic Review* **75**(4), 676–690.
- Pissarides, C. A. (2000), *Equilibrium Unemployment Theory*, The MIT Press, Cambridge, Massachusetts.
- Pissarides, C. A. (2009), ‘The unemployment volatility puzzle: Is wage stickiness the answer?’, *Econometrica*. Forthcoming.

- Ravn, M. O. and Simonelli, S. (2008), ‘Labor market dynamics and the business cycle: Structural evidence for the United States’, *Scandinavian Journal of Economics* **109**(4), 743–777.
- Schmitt-Grohe, S. and Uribe, M. (2004), Optimal operational monetary policy in the Christiano-Eichenbaum-Evans model of the U.S. business cycle, NBER Working Paper Series No. 10724, National Bureau of Economic Research.
- Shimer, R. (2004), ‘The consequences of rigid wages in search models’, *Journal of the European Economic Association* **2**(2), 469–479.
- Shimer, R. (2005), ‘The cyclical behavior of equilibrium unemployment and vacancies’, *American Economic Review* **95**(1), 25–49.
- Shimer, R. (2007), Reassessing the ins and outs of unemployment, NBER Working Paper Series No. 13421, National Bureau of Economic Research.
- Sigrist, O. (2009), European job finding rates: Evidence from the Euroepan Community Household Panel, Master’s thesis, University of Basel.
- Sims, C. A. (1992), ‘Interpreting the macroeconomic time series facts: The effects of monetary policy’, *European Economic Review* **36**(5), 975–1000.
- Smets, F. and Wouters, R. (2003), ‘An estimated dynamic stochastic general equilibrium model of the Euro area’, *Journal of the European Economic Association* **1**(5), 1123–1175.
- Smets, F. and Wouters, R. (2007), ‘Shocks and frictions in US business cycles: A Bayesian DSGE approach’, *American Economic Review* **97**(3), 586–606.
- Svejnar, J. (1986), ‘Bargaining power, fear of disagreement, and wage settlements: Theory and evidence from U.S. industry’, *Econometrica* **54**(5), 1055–1078.
- Taylor, J. B. (1993), ‘Discretion versus policy rules in practice’, *Carnegie-Rochester Conference Series on Public Policy* **39**(1), 195–214.
- Trigari, A. (2006), The role of search frictions and bargaining for inflation dynamics, IGIER Working Papers No. 304, Bocconi University.
- Uhlig, H. (2004), ‘Do technology shocks lead to a fall in total hours worked?’, *Journal of the European Economic Association* **2**(2-3), 361–371.
- van Ours, J. and Ridder, G. (1992), ‘Vacancies and the recruitment of new employees’, *Journal of Labor Economics* **10**(2), 138–155.
- Walsh, C. E. (2005), ‘Labor market search, sticky prices, and interest rate policies’, *Review of Economic Dynamics* **8**(4), 829–849.